IAS
PREVIOUS YEARS QUESTIONS (2020-1983)
SEGMENT-WISE

ORDINARY DIFFERENTIAL EQUATIONS

2020

❖ Solve the following differential equation :

$$x\cos\left(\frac{y}{x}\right)(y\,dx + x\,dy) = y\sin\left(\frac{y}{x}\right)x\,dy - y\,dx$$

[10]

❖ Find the orthogonal trajectories of the family of circles passing through the points (0, 2) and (0, –2). **[10]**

❖ Using the method of variation of parameters, solve the differential equation $y'' + (1 - \cot x)y' - y\cot x = \sin^2 x$, if $y = e^{-x}$ is one solution of CF. **[20]**

❖ Using Laplace transform, solve the initial value problem, $ty'' + 2ty' + 2y = 2$; $y(0) = 1$ and $y'(0)$ is arbitrary. Does this problem have a unique solution ? **[10]**

❖ (i) Solve the following differential equation : $(x + 1)^2 y'' - 4(x + 1)y' + 6y = 6(x + I)^2 + \sin\log(x+1)$ **[10]**

(ii) Find the general and singular solutions of the differential equation $9p^2(2 - y)^2 = 4(3 - y)$, where $p = \dfrac{dy}{dx}$. **[10]**

2019

❖ Solve the differential equations $(2y\sin x + 3y^4\sin x\cos x)\,dx - (4y^3\cos^2 x + \cos x)\,dy = 0$ **[10]**

❖ Determine the complete solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3x^2e^{2x}\sin 2x.$$ **[10]**

❖ (i) Solve the differential equation

$$\frac{d^2y}{dx^2} + (3\sin x - \cot x)\frac{dy}{dx} + 2y\sin^2 x = e^{-\cos x}\cdot\sin^2 x$$

[10]

(ii) Find the Laplace transforms of $t^{-1/2}$ and $t^{1/2}$. Prove that the Laplace transform of $t^{n+\frac{1}{2}}$, where $n \in \mathbb{N}$, is

$$\frac{\Gamma\left(n+1+\frac{1}{2}\right)}{s^{n+1+\frac{1}{2}}}$$

[10]

❖ Find the linearly independent solutions of the corresponding homogeneous differential equation of the equation $x^2 y'' - 2xy' + 2y = x^3\sin x$ and then find the general solution of the given equation by the method of variation of parameters. **[15]**

❖ Obtain the singular solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2\left(\frac{y}{x}\right)^2\cot^2\alpha - 2\left(\frac{dy}{dx}\right)\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2\operatorname{cosec}^2\alpha = 1$$

Also find the complete primitive of the given differential equation. Give the geometrical interpretations of the complete primitive and singular solution. **[15]**

2018

❖ Solve : $y'' - y = x^2e^{2x}$ **(10)**

❖ Solve $y''' - 6y'' + 12y' - 8y = 12e^{2x} + 27e^{-x}$ **(10)**

❖ Find the Laplace transform of $f(t) = \dfrac{1}{\sqrt{t}}$. **(05)**

❖ Find the inverse Laplace transform of $\dfrac{5s^2 + 3s - 16}{(s-1)(s-2)(s+3)}$ **(05)**

❖ Solve $\left(\dfrac{dy}{dx}\right)^2 y + 2\dfrac{dy}{dx}x - y = 0$. **(13)**

❖ Solve $y'' + 16y = 32\sec 2x$. **(13)**

❖ Solve $(1+x)^2 y'' + (1+x)y' + y = 4\cos(\log(1+x))$ **(13)**

❖ Solve the initial value problem **(13)**

$$y'' - 5y' + 4y = e^{2t}$$

$$y(0) = \frac{19}{12},\ y'(0) = \frac{8}{3}$$

- Find α and β such that $x^\alpha y^\beta$ is an integrating factor of $(4y^2+3xy)dx - (3xy+2x^2)dy = 0$ and solve the equation. **(12)**
- Find $f(y)$ such that $(2xe^y+3y^2)dy+(3x^2+f(y))dx=0$ is exact and hence solve. **(12)**

2017

- Find the differential equation representing all the circles in the x-y plane. **(10)**
- Suppose that the streamlines of the fluid flow are given by a family of curves xy = c. Find the equipotential lines, that is, the orthogonal trajectories of the family of curves representing the streamlines. **(10)**
- Solve the following simultaneous linear differential equations: $(D+1)y = z + e^x$ and $(D+1)z = y + e^x$ where y and z are functions of independent variable x and $D \equiv \frac{d}{dx}$. **(08)**
- If the growth rate of the population of bacteria at any time t is proportional to the amount present at that time and population doubles in one week, then how much bacterias can be expected after 4 weeks? **(08)**
- Consider the differential equation $xy\,p^2-(x^2+y^2-1)p+xy=0$ where $p=\frac{dy}{dx}$. Substituting $u = x^2$ and $v = y^2$ reduce the equation to Clairaut's form in terms of u, v and $p'=\frac{dv}{du}$. Hence, or otherwise solve the equation. **(10)**
- Solve the following initial value differential equations: 20y''+4y'+y=0, y(0)=3.2 and y'(0)=0. **(07)**
- Solve the differential equation: $$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin(x^2).$$ **(09)**
- Solve the following differential equation using method of variation of parameters: $$\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=44-76x-48x^2.$$ **(08)**
- Solve the following initial value problem using Laplace transform: $$\frac{d^2y}{dx^2}+9y=r(x),\ y(0)=0,\ y'(0)=4$$ where $r(x)=\begin{cases}8\sin x & \text{if } 0<x<\pi\\ 0 & \text{if } x\geq\pi\end{cases}$ **(17)**

2016

- Find a particular integral of $$\frac{d^2y}{dx^2}+y=e^{x/2}\sin\frac{x\sqrt{3}}{2}.$$ **(10)**
- Solve : $$\frac{dy}{dx}=\frac{1}{1+x^2}\left(e^{\tan^{-1}x}-y\right)$$ **(10)**
- Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self-orthogonal. **(10)**
- Solve : $\{y(1-x\tan x)+x^2\cos x\}\,dx - x\,dy = 0$ **(10)**
- Using the method of variation of parameters, solve the differential equation $$(D^2+2D+1)y=e^{-x}\log(x),\ \left[D=\frac{d}{dx}\right]$$ **(15)**
- Find the general solution of the equation $$x^2\frac{d^3y}{dx^3}-4x\frac{d^2y}{dx^2}+6\frac{dy}{dx}=4.$$ **(15)**
- Using Laplace Transformation, solve the following : $y''-2y'-8y=0,\ y(0)=3,\ y'(0)=6$ **(10)**

2015

- Solve the differential equation : $$x\cos x\frac{dy}{dx}+y(x\sin x+\cos x)=1.$$ **(10)**
- Solve the differential equation : $$(2xy^4e^y+2xy^3+y)\,dx+(x^2y^4e^y-x^2y^2-3x)dy=0$$ **(10)**
- Find the constant a so that $(x+y)^a$ is the Integrating factor of $(4x^2+2xy+6y)dx+(2x^2+9y+3x)dy=0$ and hence solve the differential eauation. **(12)**
- (i) Obtain Laplace Inverse transform of $$\left\{\ell n\left(1+\frac{1}{s^2}\right)+\frac{s}{s^2+25}e^{-\pi s}\right\}.$$
  (ii) Using Laplace transform, solve $y''+y=t,\ y(0)=1,\ y'(0=-2.$ **(12)**
- Solve the differential equation $x=py-p^2$ where $p=\frac{dy}{dx}$
- Solve : $$x^4\frac{d^4y}{dx^4}+6x^3\frac{d^3y}{dx^3}+4x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^2+2\cos(\log_e x).$$

2014

- Justify that a differential equation of the form : $[y + x f(x^2 + y^2)]\, dx+[y f(x^2 + y^2) -x]\, dy = 0$, where $f(x^2 + y^2)$ is an arbitrary function of (x^2+y^2), is not an exact differential equation and $\frac{1}{x^2+y^2}$ is an integrating factor for it. Hence solve this differential equation for $f(x^2+y^2) = (x^2+y^2)^2$. **(10)**
- Find the curve for which the part of the tangent cut-off by the axes is bisected at the point of tangency. **(10)**
- Solve by the method of variation of parameters : $\frac{dy}{dx} -5y = \sin x$ **(10)**
- Solve the differential equation : **(20)**
  $$x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\log_e x)$$
- Solve the following differential equation : **(15)**
  $$x\frac{d^2y}{dx^2}-2(x+1)\frac{dy}{dx}+(x+2)y=(x-2)e^{2x},$$
  when e^x is a solution to its corresponding homogeneous differential equation.
- Find the sufficient condition for the differential equation $M(x, y)\, dx + N(x, y)\, dy = 0$ to have an integrating factor as a function of $(x+y)$. What will be the integrating factor in that case? Hence find the integrating factor for the differential equation $(x^2 + xy)\, dx + (y^2+xy)\, dy = 0$ and solve it. **(15)**
- Solve the initial value problem
  $$\frac{d^2y}{dt^2}+y=8e^{-2t}\sin t,\ y(0)=0, y'(0)=0$$
  by using Laplace-transform

2013

- y is a function of x, such that the differential coefficient $\frac{dy}{dx}$ is equal to $\cos(x + y) + \sin(x + y)$. Find out a relation between x and y, which is free from any derivative/differential. **(10)**
- Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^n = a \sin n\theta$, (r, θ) being the plane polar coordinates. **(10)**
- Solve the differential equation $(5x^3 + 12x^2 + 6y^2)\, dx + 6xydy = 0$. **(10)**
- Using the method of variation of parameters, solve the differential equation $\frac{d^2y}{dx^2} + a^2y = \sec ax$. **(10)**
- Find the general solution of the equation **(15)**
  $$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \ln x \sin(\ln x).$$
- By using Laplace transform method, solve the differential equation $(D^2 + n^2)\, x = a \sin(nt + \alpha)$, $D^2 = \frac{d^2}{dt^2}$ subject to the initial conditions $x = 0$ and $\frac{dx}{dt} = 0$, at $t = 0$, in which a, n and α are constants. **(15)**

2012

- Solve $\frac{dy}{dx}=\frac{2xy\,e^{(x/y)^2}}{y^2(1+e^{(x/y)^2})+2x^2e^{(x/y)^2}}$ **(12)**
- Find the orthogonal trajectories of the family of curves $x^2+y^2=ax$. **(12)**
- Using Laplace transforms, solve the intial value problem $y''+2y'+y=e^{-t}, y(0)=-1, y'(0)=1$ **(12)**
- Show that the differential equation
  $$(2xy\log y)dx+\left(x^2+y^2\sqrt{y^2+1}\right)dy=0$$
  is not exact. Find an integrating factor and hence, the solution of the equation. **(20)**
- Find the general solution of the equation $y'''-y''=12x^2+6x.$ **(20)**
- Solve the ordinary differential equation $x(x-1)y''-(2x-1)y'+2y=x^2(2x-3)$ **(20)**

2011

- Obtain the solution of the ordinary differential equation $\frac{dy}{dx}=(4x+y+1)^2$, if y(0) = 1. **(10)**
- Determine the orthogonal trajectory of a family of curves represented by the polar equation r = a (1 – cosθ,) (r, θ) being the plane polar coordinates of any point. **(10)**
- Obtain Clairaut's orm of the differential equation $\left(x\frac{dy}{dx}-y\right)\left(y\frac{dy}{dx}+y\right)=a^2\frac{dy}{dx}$. Also find its general solution. **(15)**
- Obtain the general solution of the second order ordinary differential equation $y''-2y'+2y=x+e^x\cos x,$ where dashes denote derivatives w.r. to x. **(15)**
- Using the method of variation of parameters, solve the second order differedifferential equation $\frac{d^2y}{dx^2}+4y=\tan 2x.$ **(15)**

- Use Laplace transform method to solve the following initial value problem:
  $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$, $x(0) = 2$ and $\left.\frac{dx}{dt}\right|_{t=0} = -1$ **(15)**

2010

- Consider the differential equation
  $y' = \alpha x, x > 0$
  where α is a constant. Show that–
  (i) if $\phi(x)$ is any solution and $\Psi(x) = \phi(x)\, e^{-\alpha x}$, then $\Psi(x)$ is a constant;
  (ii) if $\alpha < 0$, then every solution tends to zero as $x \to \infty$. **(12)**
- Show that the differential equation
  $(3y^2 - x) + 2y(y^2 - 3x)y' = 0$
  admits an integrating factor which is a function of $(x+y^2)$. Hence solve the equation. **(12)**
- Verify that
  $\frac{1}{2}(Mx+Ny)\,d(\log_e(xy)) + \frac{1}{2}(Mx-Ny)\,d(\log_e(\tfrac{x}{y}))$
  $= M\,dx + N\,dy$
  Hence show that–
  (i) if the differential equation $M\,dx + N\,dy = 0$ is homogeneous, then $(Mx+Ny)$ is an integrating factor unless $Mx + Ny \equiv 0$;
  (ii) if the differential equation $Mdx + Ndy = 0$ is not exact but is of the form
  $f_1(xy)y\,dx + f_2(xy)x\,dy = 0$
  then $(Mx - Ny)^{-1}$ is an integrating factor unless $Mx - Ny \equiv 0$. **(20)**
- Show that the set of solutions of the homogeneous linear differential equation
  $y' + p(x)y = 0$
  on an interval $I = [a, b]$ forms a vector subspace W of the real vector space of continuous functions on I. what is the dimension of W?. **(20)**
- Use the method of undetermined coefficients to find the particular solution of
  $y'' + y = \sin x + (1 + x^2)e^x$
  and hence find its general solution. **(20)**

2009

- Find the Wronskian of the set of functions
  $\{3x^3, |3x^3|\}$
  on the interval [–1, 1] and determine whether the set is linearly dependent on [–1, 1]. **(12)**
- Find the differential equation of the family of circles in the xy-plane passing through (–1, 1) and (1, 1). **(20)**
- Find the inverse Laplace transform of
  $F(s) = \ln\left(\frac{s+1}{s+5}\right)$. **(20)**
- Solve: $\frac{dy}{dx} = \frac{y^2(x-y)}{3xy^2 - x^2y - 4y^3}, y(0) = 1$. **(20)**

2008

- Solve the differential equation
  $ydx + (x + x^3y^2)dy = 0$. **(12)**
- Use the method of variation of parameters to find the general solution of $x^2x'' - 4xy' + 6y = -x^4\sin x$. **(12)**
- Using Laplace transform, solve the initial value problem $y'' - 3y' + 2y = 4t + e^{3t}$ with $y(0) = 1$, $y'(0) = -1$. **(15)**
- Solve the differential equation
  $x^3y'' - 3x^2y' + xy = \sin(\ln x) + 1$. **(15)**
- Solve the equation $y - 2xp + yp^2 = 0$ where $p = \frac{dy}{dx}$. **(15)**

2007

- Solve the ordinary differential equation
  $\cos 3x\frac{dy}{dx} - 3y\sin 3x = \frac{1}{2}\sin 6x + \sin^2 3x$,
  $0 < x < \frac{\pi}{2}$. **(12)**
- Find the solution of the equation
  $\frac{dy}{y} + xy^2dx = -4xdx$. **(12)**
- Determine the general and singular solutions of the equation $y = x\frac{dy}{dx} + a\frac{dy}{dx}\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{-1/2}$ 'a' being a constant. **(15)**
- Obtain the general solution of $\left[D^3 - 6D^2 + 12D - 8\right]$
  $y = 12\left(e^{2x} + \frac{9}{4}e^{-x}\right)$, where $D = \frac{d}{dx}$. **(15)**

- Solve the equation $2x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 3y = x^3$. **(15)**
- Use the method of variation of parameters to find the general solution of the equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^x$. **(15)**

2006

- Find the family of curves whose tangents form an angle $\pi/4$ with the hyperbolas xy = c, c > 0. **(12)**
- Solve the differential equation $\left(xy^2 + e^{-1/x^3}\right)dx - x^2y\,dy = 0$. **(12)**
- Solve $\left(1+y^2\right) + \left(x - e^{-\tan^{-1}y}\right)\frac{dy}{dx} = 0$. **(15)**
- Solve the equation $x^2p^2 + yp(2x+y) + y^2 = 0$ using the substituion y = u and xy=v and find its singular solution, where $p = \frac{dy}{dx}$. **(15)**
- Solve the differential equation $x^2\frac{d^3y}{dx^3} + 2x\frac{d^2y}{dx^2} + 2\frac{y}{x} = 10\left(1 + \frac{1}{x^2}\right)$ **(15)**
- Solve the differential equation $\left(D^2 - 2D + 2\right)y = e^x \tan x$, where $D = \frac{d}{dx}$, by the method of variation of parameters. **(15)**

2005

- Find the orthogonal trajectory of a system of co-axial circles $x^2+y^2+2gx+c=0$, where g is the parameter. **(12)**
- Solve $xy\frac{dy}{dx} = \sqrt{x^2 - y^2 - x^2y^2 - 1}$. **(12)**
- Solve the differential equation $(x+1)^4 D^3 + 2(x+1)^3 D^2 - (x+1)^2 D + (x+1)y = \frac{1}{x+1}$. **(15)**
- Solve the differential equation $(x^2+y^2)(1+p)^2 - 2(x+y)(1+p)(x+yp) + (x+yp)^2 = 0$, where $p = \frac{dy}{dx}$, by reducing it to Clairaut's form by using suitable substitution. **(15)**
- Solve the differential equation (sin x–x cos x) $y'' - x\sin xy' + y\sin x = 0$ given that y = sin x is a solution of this equation. **(15)**
- Solve the differential equation $x^2y'' - 2xy' + 2y = x\log x, x > 0$ by variation of parameters. **(15)**

2004

- Find the solution of the following differential equation $\frac{dy}{dx} + y\cos x = \frac{1}{2}\sin 2x$. **(12)**
- Solve $y(xy+2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0$. **(12)**
- Solve $\left(D^4 - 4D^2 - 5\right)y = e^x(x + \cos x)$. **(15)**
- Reduce the equation (px–y) (py+x) = 2p where $p = \frac{dy}{dx}$ to Clairaut's equation and hence solve it. **(15)**
- Solve $(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (x+1)e^x$. **(15)**
- Solve the following differential equation $(1-x^2)\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} - (1+x^2)y = x$. **(15)**

2003

- Show that the orthogonal trajectory of a system of confocal ellipses is self orthogonal. **(12)**
- Solve $x\frac{dy}{dx} + y\log y = xye^x$. **(12)**
- Solve $(D^5 - D)y = 4(e^x + \cos x + x^3)$, where $D = \frac{d}{dx}$. **(15)**
- Solve the differential equation $(px^2 + y^2)(px + y) = (p+1)^2$ where $p = \frac{dy}{dx}$, by reducing it to Clairaut's form using suitable substitutions. **(15)**
- Solve $\left(1+x\right)^2 y'' + \left(1+x\right)y' + y = \sin 2\left[\log\left(1+x\right)\right]$. **(15)**
- Solve the differential equation $x^2y'' - 4xy' + 6y = x^4\sec^2 x$ by variation of parameters. **(15)**

2002

- Solve $x\frac{dy}{dx} + 3y = x^3y^2$. **(12)**

❖ Find the values of λ for which all solutions of $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-\lambda y=0$ tend to zero as $x\to\infty$. **(12)**

❖ Find the value of constant λ such that the following differential equation becomes exact.

$$\left(2xe^y+3y^2\right)\frac{dy}{dx}+\left(3x^2+\lambda e^y\right)=0$$

Further, for this value of λ, solve the equation. **(15)**

❖ Solve $\frac{dy}{dx}=\frac{x+y+4}{x-y-6}$. **(15)**

❖ Using the method of variation of parameters, find the solution of $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$ with y(0) = 0 and $\left(\frac{dy}{dx}\right)_{x=0}=0$. **(15)**

❖ Solve (D–1) (D^2–2 D+2) y = e^x where $D=\frac{d}{dx}$. **(15)**

2001

❖ A continuous function y(t) satisfies the differential equation $\frac{dy}{dt}=\begin{cases}1+e^{1-t}, 0\le t<1\\ 2+2t-3t^2, 1\le t\le 5\end{cases}$

If y(0) = –e, find y(2). **(12)**

❖ Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\log_e x$. **(12)**

❖ Solve $\frac{dy}{dx}+\frac{y}{x}\log_e y=\frac{y(\log_e y)^2}{x^2}$. **(15)**

❖ Find the general solution of $ayp^2+(2x-b)p-y=0$, a>o. **(15)**

❖ Solve $(D^2+1)^2$ y = 24x cos x given that y=Dy=D^2y=0 and D^3y = 12 when x = 0. **(15)**

❖ Using the method of variation of parameters, solve $\frac{d^2y}{dx^2}+4y=4\tan 2x$. **(15)**

2000

❖ Show that $3\frac{d^2y}{dx^2}+4x\frac{dy}{dx}-8y=0$ has an integral which is a polynomial in x. Deduce the general solution. **(12)**

❖ Reduce $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$, where P, Q, R are functions of x, to the normal form. Hence solve

$$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin 2x. \quad \textbf{(15)}$$

❖ Solve the differential equation y = x–2a p+ap^2. Find the singular solution and interpret it geometrically. **(15)**

❖ Show that (4x+3y+1)dx+(3x+2y+1) dy = 0 represents a family of hyperbolas with a common axis and tangent at the vertex. **(15)**

❖ Solve $x\frac{dy}{dx}-y=(x-1)\left(\frac{d^2y}{dx^2}-x+1\right)$ by the method of variation of parameters. **(15)**

1999

❖ Solve the differential equation

$$\frac{xdx+ydy}{xdy-ydx}=\left(\frac{1-x^2-y^2}{x^2+y^2}\right)^{1/2}$$

❖ Solve $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4\frac{dy}{dx}-2y=e^x+\cos x$.

❖ By the method of variation of parameters solve the differential equation $\frac{d^2y}{dx^2}+a^2y=\sec(ax)$.

1998

❖ Solve the differential equation $xy-\frac{dy}{dx}=y^3e^{-x^2}$

❖ Show that the equation (4x+3y+1) dx + (3x+2y+1) dy = 0 represents a family of hyperbolas having as asymptotes the lines x+y = 0; 2x+y+1=0. **(1992)**

❖ Solve the differential equation y = 3px + 4p^2.

❖ Solve $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=e^{4x}(x^2+9)$.

❖ Solve the differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x\sin x.$$

1997

❖ Solve the initial value problem $\frac{dy}{dx}=\frac{x}{x^2y+y^3}$, y(0) = 0.

- Solve $(x^2-y^2+3x-y)\,dx + (x^2-y^2+x-3y)dy = 0$.
- Solve $\frac{d^4y}{dx^4}+6\frac{d^3y}{dx^3}+11\frac{d^2y}{dx}+6\frac{dy}{dx}=20e^{-2x}\sin x$
- Make use of the transformation $y(x) = u(x)\sec x$ to obtain the solution of $y''-2y'\tan x+5y=0$; $y(0) = 0$; $y'(0)=\sqrt{6}$.
- Solve $(1+2x)^2\frac{d^2y}{dx^2}-6\ (1+2x)\ \frac{dy}{dx}+16y\ = 8\ (1+2x)^2$; $y(0)=0$ and $y'(0)=2$.

1996

- Solve $x^2(y-px) = yp^2;\left(p=\frac{dy}{dx}\right)$.
- Solve $y\sin 2x\,dx - (1+y^2+\cos^2 x)\,dy = 0$.
- Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}$ + 10y + 37 sin 3x = 0. Find the value of y when $x=\pi/2$, if it is given that y = 3 and $\frac{dy}{dx}=0$ when x = 0.
- Solve $\frac{d^4y}{dx^4}+2\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}=x^2+3e^{2x}+4\sin x$.
- Solve $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x+\log x$.

1995

- Solve $(2x^2+3y^2-7)x\,dx-(3x^2+2y^2-8)\,y\,dy=0$.
- Test whether the equation $(x+y)^2\,dx-(y^2-2xy-x^2)\,dy=0$ is exact and hence solve it.
- Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$.**(1998)**
- Determine all real valued solutions of the equation $y'''-iy''+y'-iy=0$, $y'=\frac{dy}{dx}$.
- Find the solution of the equation $y''+4y=8\cos 2x$ given that y = 0 and $y'=2$ when x = 0.

1994

- Solve $\frac{dy}{dx}$ + x sin 2y = $x^3\cos^2 y$.
- Show that if $\frac{1}{Q}\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)$ is a function of x only say, f(x), then $F(x)=e^{\int f(x)dx}$ is an integrating factor of Pdx + Qdy = 0.
- Find the family of curves whose tangent form angle $\pi/4$ with the hyperbola xy = c.
- Transform the differential equation $\frac{d^2y}{dx^2}\cos x+\frac{dy}{dx}\sin x-2y\cos^3 x=2\cos^5 x$ into one having z an independent variable where z = sin x and solve it.
- If $\frac{d^2x}{dt^2}+\frac{g}{b}(x-a)=0$, (a, b and g being positive constants) and x = a′ and $\frac{dx}{dt}=0$ when t = 0, show that $x=a+(a'-a)\cos\sqrt{\frac{g}{b}}t$.
- Solve $(D^2-4D+4)\,y = 8x^2e^{2x}\sin 2x$, where, $D=\frac{d}{dx}$.

1993

- Show that the system of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self orthogonal.
- Solve $\left\{y\left(1+\frac{1}{x}\right)+\cos y\right\}dx+\{x+\log x\ \square x\sin y\}$ dy = 0.
- Solve $\frac{d^2y}{dx^2}+w_0^2y$ = a coswt and discuss the nature of solution as $w\to w_0$.
- Solve $(D^4+D^2+1)\,y = e^{-x/2}\cos\left(\frac{\sqrt{3}x}{2}\right)$.

1992

- By eliminating the constants a, b obtain the differential equation of which $xy = ae^x+be^{-x}+x^2$ is a solution.
- Find the orthogonal trajectories of the family of semicubical parabolas $ay^2=x^3$, where a is a variable parameter.

- Show that $(4x+3y+1)\,dx + (3x+2y+1)\,dy = 0$ represents hyperbolas having the following lines as asymptotes
  $x + y = 0$, $2x+y+1 = 0$. **(1998)**
- Solve the following differential equation $y(1+xy)\,dx+x(1-xy)\,dy = 0$.
- Solve the following differential equation $(D^2+4)y = \sin 2x$ given that when $x = 0$ then $y = 0$ and $\frac{dy}{dx} = 2$.
- Solve $(D^3-1)y = xe^x + \cos^2 x$.
- Solve $(x^2D^2 + xD - 4)\,y = x^2$.

1991

- If the equation $Mdx + Ndy = 0$ is of the form $f_1(xy).\,ydx + f_2(xy).\,x\,dy = 0$, then show that $\frac{1}{Mx-Ny}$ is an integrating factor provided $Mx - Ny \neq 0$.
- Solve the differential equation.
  $(x^2-2x+2y^2)\,dx + 2xy\,dy = 0$.
- Given that the differential equation $(2x^2y^2 + y)\,dx - (x^3y-3x)\,dy = 0$ has an integrating factor of the form $x^h y^k$, find its general solution.
- Solve $\frac{d^4y}{dx^4} - m^4 y = \sin mx$.
- Solve the differential equation
  $\frac{d^4y}{dx^4} - 2\frac{d^3y}{dx^3} + 5\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 4y = e^x$.
- Solve the differential equation
  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y = xe^{-x}$, given that $y = 0$ and $\frac{dy}{dx} = 0$, when $x = 0$.

1990

- If the equation $\lambda^n+a_1\lambda^{n-1}+.........+a_n = 0$ (in unknown λ) has distinct roots $\lambda_1,\lambda_2, ... \lambda_n$. Show that the constant coefficients of differential equation
  $\frac{d^ny}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + ... + a_{n-1}\frac{dy}{dx} + a_n = b$ has the most general solution of the form
  $y = c_0(x) + c_1e^{\lambda_1 x} + c_2e^{\lambda_2 x} + ... + c_ne^{\lambda_n x}$.
  where $c_1, c_2 .... c_n$ are parameters. what is $c_0(x)$?
- Analyses the situation where the λ – equation in (a) has repeated roots.
- Solve the differential equation
  $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = 0$ is explicit form. If your answer contains imaginary quantities, recast it in a form free of those.
- Show that if the function $\frac{1}{t-f(t)}$ can be integrated (w.r.t 't'), then one can solve $\frac{dy}{dx} = f(y/x)$, for any given f. Hence or otherwise.
  $\frac{dy}{dx} + \frac{x-3y+2}{3x-y+6} = 0$
- Verify that $y = (\sin^{-1}x)^2$ is a solution of $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$. Find also the most general solution.

1989

- Find the value of y which satisfies the equation $(xy^3 - y^3 - x^2e^x) + 3xy^2\frac{dy}{dx} = 0$; given that $y=1$ when $x = 1$.
- Prove that the differential equation of all parabolas lying in a plane is $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)^{-2/3} = 0$.
- Solve the differential equation
  $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = 1 + x^2$.

1988

- Solve the differential equation
  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 2e^x \sin x$.
- Show that the equation $(12x+7y+1)\,dx + (7x+4y+1)\,dy = 0$ represents a family of curves having as asymptotes the lines $3x+2y-1=0$, $2x+y+1=0$.
- Obtain the differential equation of all circles in a plane n the form $\frac{d^3y}{dx^3}\left\{1+\left(\frac{dy}{dx}\right)^2\right\} - 3\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right)^2 = 0$.

1987

- Solve the equation $x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} = y + e^x$
- If $f(t) = t^{p-1}$, $g(t) = t^{q-1}$ for $t > 0$ but $f(t) = g(t) = 0$ for $t \leq 0$, and $h(t) = f * g$, the convolution of f, g

show that $h(t)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}t^{p+q-1};t\geq 0$ and p, q are positive constants. Hence deduce the formula $B(p,q)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.

1985

- Consider the equation $y'+5y=2$. Find that solution ϕ of the equation which satisfies $\phi(1)=3\phi'(0)$.
- Use Laplace transform to solve the differential equation $x''-2x'+x=e^t,\left('=\frac{d}{dt}\right)$ such that $x(0)=2,x'(0)=-1$.
- For two functions f, g both absolutely integrable on $(-\infty,\infty)$, define the convolution f * g.

  If L(f), L(g) are the Laplace transforms of f, g show that L (f * g) = L(f). L(g).
- Find the Laplace transform of the function $f(t)=\begin{cases}1 & 2n\pi\leq t<(2n+1)\pi\\ -1 & (2n+1)\pi\leq t\leq(2n+2)\pi\end{cases}$

  n = 0, 1, 2, ...

1984

- Solve $\frac{d^2y}{dx^2}+y=\sec x$.
- Using the transformation $y=\frac{u}{x^k}$, solve the equation $xy''+(1+2k)y'+xy=0$.
- Solve the equation $(D^2+1)x=t\cos 2t$, given that $x_0=x_1=0$ by the method of Laplace transform.

1983

- Solve $x\frac{d^2y}{dx}+(x-1)\frac{dy}{dx}-y=x^2$.
- Solve $(y^2+yz)\,dx+(xz+z^2)\,dy+(y^2-xy)\,dz=0$.
- Solve the equation by the method of Laplace transform, given that y = –3 when t = 0, y = –1 when t = 1.

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IFoS
PREVIOUS YEARS QUESTIONS (2020-2000)
SEGMENT-WISE

ORDINARY DIFFERENTIAL EQUATIONS
(ACCORDING TO THE NEW SYLLABUS PATTERN) PAPER - II

2020

❖ Solve the initial value problem : $(2x^2 + y)\,dx + (x^2y - x)\,dy = 0$, $y(1) = 2$. **[08]**

❖ Solve the differential equation
$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 16x - 12e^{2x}.$$ **[08]**

❖ Find one solution of the differential equation
$$\left(x^2+1\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$
by inspection and using that solution determine the other linearly independent solution of the given equation. Obtain the general solution of the given differential equation. **[10]**

❖ Solve the differential equation
$$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}.$$ **[15]**

❖ Reduce the differential equation
$$xp^2 - 2yp + x + 2y = 0, \left(p = \frac{dy}{dx}\right),$$
to Clairaut's form and obtain its complete primitive Also, determine a singular solution of the given differential equation. **[15]**

2019

❖ Solve the differential equation
$$\left(D^2+1\right)y = x^2\sin 2x; D \equiv \frac{d}{dx}.$$ **(08)**

❖ Solve the differential equation $(px - y)(py + x) = h^2 p$, where $p = y'$. **(08)**

❖ Solve by the method of variation parameters the differential equation
$$x''(t) - \frac{2x(t)}{t^2} = t, \quad \text{where } 0 < t < \infty$$ **(15)**

❖ Find the general solution of the differential equation
$$\ddot{x} + 4x = \sin^2 2t$$
Hence find the particular solution satisfying the conditions
$$x\left(\frac{\pi}{8}\right) = 0 \text{ and } \dot{x}\left(\frac{\pi}{8}\right) = 0$$ **(15)**

❖ Find the general solution of the differential equation
$$(x-2)y'' - (4x-7)y' + (4x-6)y = 0$$ **(10)**

2018

❖ Find the complementary function and particular integral for the equation
$$\frac{d^2y}{dx^2} - y = xe^x + \cos^2 x$$ and hence the general solution of the equation. **(08)**

❖ Solve $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = xe^x \log x (x > 0)$ by the method of variation of parameters. **(08)**

❖ Solve the differential equation
$$(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0.$$ **(10)**

❖ Solve $\dfrac{dy}{dx} = \dfrac{4x+6y+5}{3y+2x+4}$ **(10)**

❖ A snowball of radius $r(t)$ melts at a uniform rate. If half of the mass of the snowball melts in one hour, how much time will it take for the entire mass of the snowball to melt, correct to two decimal places? Conditions remain unchanged for the entire process. **(15)**

2017

❖ Solve $(2D^3 - 7D^2 + 7D - 2)\,y = e^{-8x}$ where $D = \dfrac{d}{dx}$. **(8)**

❖ Solve the differential equation
$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4.$$ **(8)**

❖ Solve the differential equation
$$\left(\frac{dy}{dx}\right)^2 + 2\cdot\frac{dy}{dx}\cdot y\cot x = y^2.$$ **(15)**

❖ Solve the differential equation
$$e^{3x}\left(\frac{dy}{dx} - 1\right) + \left(\frac{dy}{dx}\right)^3 e^{2y} = 0.$$ **(10)**

❖ Solve $\frac{d^2y}{dx^2}+4y=\tan 2x$ by using the method of variation of parameter. **(10)**

2016

❖ Obtain the curve which passes through (1, 2) and has a slope $=\frac{-2xy}{x^2+1}$. Obtain one asymptote to the curve. **(8)**

❖ Solve the dE to get the particular integral of $\frac{d^4y}{dx^4}+2\frac{d^2y}{dx^2}+y=x^2\cos x$. **(8)**

❖ Using the method of variation of parameters, solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2e^x$. **(10)**

❖ Obtain the singular solution of the differential equation

$y^2-2pxy+p^2(x^2-1)=m^2,\ p=\frac{dy}{dx}$. **(10)**

❖ Solve the differential equation **(10)**

$\frac{dy}{dx}-y=y^2(\sin x+\cos x)$.

2015

❖ Reduce the differential equation

$x^2p^2+yp(2x+y)+y^2=0,\ p=\frac{dy}{dx}$ to Clairaut's form. Hence, find the singular solution of the equation. **(8)**

❖ Solve the differential equation

$x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=\frac{1}{(1-x)^2}$ **(8)**

❖ Solve $x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin x^2$ by changing the independent variable. **(10)**

❖ Solve $(D^4+D^2+1)y=e^{-x/2}\cos\left(\frac{x\sqrt{3}}{2}\right)$,

where $D=\frac{d}{dx}$. **(10)**

2014

❖ Solve the differential equation

$y=2px+p^2y,\ p=\frac{dy}{dx}$

and obtain the non-singular solution **(8)**

❖ Solve

$\frac{d^4y}{dx^4}-16y=x^4+\sin x$. **(8)**

❖ Solve the following differential equation

$\frac{dy}{dx}=\frac{2y}{x}+\frac{x^3}{y}+x\tan\frac{y}{x^2}$. **(10)**

❖ Solve by the method of variation of parameters

$y''+3y'+2y=x+\cos x$. **(10)**

❖ Solve the D.E.

$\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4\frac{dy}{dx}-2y=e^x+\cos x$. **(10)**

2013

❖ Solve

$\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$ **(8)**

❖ Solve the differential equation

$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin 2x$

by changing the dependent variable. **(13)**

❖ Solve

$(D^3+1)y=e^{\frac{x}{2}}\sin\left(\frac{\sqrt{3}}{2}x\right)$ where $D=\frac{d}{dx}$. **(13)**

❖ Apply the method of variation of parameters to solve

$\frac{d^2y}{dx^2}-y=2(1+e^x)^{-1}$ **(13)**

2012

❖ Solve $\frac{dy}{dx}-\frac{\tan y}{1+x}=(1+x)e^x\sec y$. ***(8)***

❖ Solve and find the singular solution of

$x^3p^2+x^2py+a^3=0$ ***(8)***

❖ Solve: $x^2y\frac{d^2y}{dx^2}+\left(x\frac{dy}{dx}-y\right)^2=0$ **(10)**

❖ Solve $\frac{d^4y}{dx^4}+2\frac{d^2y}{dx^2}+y=x^2\cos x$. **(10)**

❖ Solve $x=y\frac{dy}{dx}-\left(\frac{dy}{dx}\right)^2$ **(10)**

❖ Solve $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=(1-x)^{-2}$ **(10)**

2011

❖ Find the family of curves whose tangents form an angle π/4 With hyperbolas xy = c. **(10)**

❖ Solve $\frac{d^2y}{dx^2}-2\tan x\frac{dy}{dx}+5y=\sec x.e^x$. **(10)**

❖ Solve $p^2+2py\cot x=y^2$ Where $p=\frac{dy}{dx}$. **(10)**

❖ Solve $\left\{x^4D^4+6x^3D^3+9x^2D^2+3xD+1\right\}y=\left(1+\log x\right)^2$, Where $D=\frac{d}{dx}$. **(15)**

❖ Solve $(D^4+D^2+1)y=ax^2+be^{-x}\sin 2x$, where $D\equiv\frac{d}{dx}$ **(15)**

2010

❖ Show that cos (x + y) is an integrating factor of $y\,dx+\left[y+\tan\left(x+y\right)\right]dy=0$. Hence solve it **(8)**

❖ Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$ **(8)**

❖ Solve the following differential equation $\frac{dy}{dx}=\sin^2\left(x-y+6\right)$ **(8)**

❖ Find the general solution of $\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+\left(x^2+1\right)y=0$ **(12)**

❖ Solve $\left(\frac{d}{dx}-1\right)^2\left(\frac{d^2}{dx^2}+1\right)^2y=x+e^x$ **(10)**

❖ Solve by the method of variation of parameters the following equation $\left(x^2-1\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=\left(x^2-1\right)^2$ **(10)**

2009

❖ Solve $\sec^2 y\frac{dy}{dx}+2x\tan y=x^3$ **(10)**

❖ Find the 2nd order ODE for which e^x and x^2 e^2 are solutions. **(10)**

❖ Solve $\left(y^3-2yx^2\right)dx+\left(2xy^2-x^3\right)dy=0$. **(10)**

❖ Solve $\left(\frac{dy}{dx}\right)^2-2\frac{dy}{dx}\cos hx+1=0.$ **(08)**

❖ Solve $\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}+3\frac{dy}{dx}+y=x^2e^{-x}$ **(10)**

❖ Show that e^{x^2} is a solution of $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(4x^2-2\right)y=0$. **(12)**
Find a second independent solution.

2008

❖ Show that the functions $y_1(x)=x^2$ and $y_2(x)=x^2\log_e x$ are linearly independent obtain the differential equation that has $y_1(x)$ and $y_2(x)$ as the independent solutions. **(10)**

❖ Solve the following ordinary differential equation of the second degree : $y\left(\frac{dy}{dx}\right)^2+\left(2x-3\right)\frac{dy}{dx}-y=0$ **(10)**

❖ Reduce the equation $\left(x\frac{dy}{dx}-y\right)\left(x-y\frac{dy}{dx}\right)=2\frac{dy}{dx}$ to clairaut's form and obtain there by the singular integral of the above equation. **(10)**

❖ Solve $\left(1+x\right)^2\frac{d^2y}{dx^2}+\left(1+x\right)\frac{dy}{dx}+y=4\cos\log_e\left(1+x\right)$ **(10)**

❖ Find the general solution of the equation $\frac{d^2y}{dx^2}-\cot x\frac{dy}{dx}-\left(1-\cot x\right)y=e^x\sin x.$ **(10)**

2007

❖ Find the orthogonal trajectories of the family of the curves $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$, λ being a parameter. **(10)**

❖ Show that e^{2x} and e^{3x} are linearly independent solutions of $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$. Find the general solution when $y\left(0\right)=0$ and $\left.\frac{dy}{dx}\right]_0=1$ **(10)**

❖ Find the family of curves whose tangents form an angle $\pi/4$ with the hyperbola $xy = c$. **(10)**

❖ Apply the method of variation of parameters to solve $(D^2 + a^2)\,y = \operatorname{cosec} ax$. **(10)**

❖ Find the general solution of $\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+3y=0$ solution of it. **(10)**

2006

❖ From $x^2+y^2+2ax+2by+c=0$, derive differential equation not containing, a, b or c. **(10)**

❖ Discuss the solution of the differential equation

$$y^2=\left[1+\left(\frac{dy}{dx}\right)^2\right]=a^2$$ **(10)**

❖ Solve $x\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}-y=e^x$ **(10)**

❖ Solve $\frac{d^4y}{dx^4}-y=x\sin x$ **(10)**

❖ Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2e^x$ **(10)**

❖ Reduce

$$xy\left(\frac{dy}{dx}\right)^2-\left(x^2+y^2+1\right)\frac{dy}{dx}+xy=0$$

to clairaut's form and find its singular solution. **(10)**

2005

❖ Form the differential equation that represents all parabolas each of which has latus rectum 4a and Whose are parallel to the x-axis. **(10)**

❖ (i) The auxiliary polynomial of a certain homogenous linear differential equation with constant coefficients in factored form is

$$P(m)=m^4(m-2)^6\left(m^2-6m+25\right)^3.$$

What is the order of the differential equation and write a general solution ?

(ii) Find the equation of the one-parameter family of parabolas given by $y^2=2cx+c^2$, C real and show that this family is self-orthogonal. **(10)**

❖ Solve and examine for singular solution the following equation

$$P^2\left(x^2-a^2\right)-2pxy+y^2-b^2=0$$ **(10)**

❖ Solve the differential equation $\frac{d^2y}{dx^2}+9y=\sec 3x$ **(10)**

❖ Given $y=x+\frac{1}{x}$ is one solution solve the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$ reduction of order method. **(10)**

❖ Find the general solution of the defferential equation $\frac{d^2y}{dx^2}-2y\frac{dy}{dx}-3y=2e^x-10\sin x$ by the method of undetermined coefficients. **(10)**

2004

❖ Determine the family of orthogonal trajectories of the family $y=x+ce^{-x}$ **(10)**

❖ Show that the solution curve satisfying $(x^2-xy)\,y'=y^3$ Where $y\to 1$ *as* $x\to\infty$, is a conic section. Identify the curve. **(10)**

❖ Solve $(1+x)^2y''+(1+x)y'+y=4\cos(\ln(1+x))$, $y(0)=1, y(e-1)=\cos 1.$ **(10)**

❖ Obtain the general solution of $y''+2y'+2y=4e^{-x}x^2\sin x$. **(10)**

❖ Find the general solution of $\left(xy^3+y\right)dx+2\left(x^2y^2+x+y^4\right)dy=0$ **(10)**

❖ Obtain the general solution of $\left(D^4+2D^3-D^2-2D\right)y=x+e^{2x}$, Where $D_y=\frac{dy}{dx}$. **(10)**

2003

❖ Find the orthogonal trajectories of the family of co-axial circles $x^2+y^2+2gx+c=0$ Where g is a parameter. **(10)**

❖ Findthethreesolutionsof $\frac{d^3y}{dx^3}-2\frac{d^2y}{dx^2}-\frac{dy}{dx}+2y=0$

Which are linearly independent on every real interval. **(10)**

❖ Solve and examine for singular solution:

$y^2-2pxy+p^2\left(x^2-1\right)=m^2.$ **(10)**

❖ Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$ **(10)**

❖ Given y = x is one solutions of

$\left(x^3+1\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$ find another linearly independent solution by reducing order and write the general solution. **(10)**

❖ Solve by the method of variation of parameters

$\frac{d^2y}{dx^2}+a^2y=\sec ax,$ a real. **(10)**

2002

❖ If $(D-a)^4e^{nx}$ is denoted by z, prove that

$z\frac{\partial z}{\partial n},\frac{\partial^2z}{\partial n^2},\frac{\partial^3z}{\partial n^3}$ all vanish when n = a. Hence show

that $e^{nx},xe^{nx},x^2e^{nx},\ x^3nx$ are all solutions of

$(D-a)^4y=0$. Here D Stands for $\frac{d}{dx}$. **(10)**

❖ Solve $4xp^2-(3x+1)^2=0$ and examine for singular solutions and extraneous loci. Interpret the results geometrically. **(10)**

❖ (i) Form the differential equation whose primitive is

$$y=A\left(\sin x+\frac{\cos x}{x}\right)+B\left(\cos x-\frac{\sin x}{x}\right)$$

(ii) Prove that the orthogonal trajectory of system of parabolas belongs to the system itself. **(10)**

❖ Using variation of parameters solve the differential equation

$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(4x^2-1\right)y=-3e^{x^2}\sin 2x.$ **(10)**

❖ (i) Solve the equation by finding an integrating factor of $(x+2)\sin y\,dx+x\cos y\,dy=0.$

(ii) Verify that $\phi(x)=x^2$ is a solution of

$y''-\frac{2}{x^2}y=0$ and find a second independent solution. **(10)**

❖ Show that the solution of $\left(D^{2n+1}-1\right)y=0,$ consists of Ae^x and n paris of terms of the form $e^{ax}\left(b_r\cos\alpha x+c_r\sin\alpha x\right),$ Where $a=\cos\frac{2\pi r}{2n+1}$

and $\alpha=\sin\frac{2\pi r}{2n+1}$, r = 1, 2 ......, n and b_r, c_r are arbitrary constants. **(10)**

2001

❖ A constant coefficient differential equation has auxiliary equation expressible in factored form as $P(m)=m^3(m-1)^2\left(m^2+2m+5\right)^2$. What is the order of the differential equation and find its general solution. **(10)**

❖ Solve $x^2\left(\frac{dy}{dx}\right)^2+y(2x+y)\frac{dy}{dx}+y^2=0$ **(10)**

❖ Using differential equations show that the system of confocal conics given by $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1,$ is self othogonal. **(10)**

❖ Solve $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$ given that

$y=e^{a\sin^{-1}x}$ is one solution of this equation. **(10)**

❖ Find a general solution $y''+y=\tan x,$

$-\pi/2<x<\pi/2$ by variation of parameters. **(10)**

2000

❖ Solve

$\left(x^2+y^2\right)(1+P)^2-2(x+y)(1+p)(x+yp)+(x+yp)^2=0$

$P\equiv\frac{dy}{dx}.$ Interpret geometrically the factors in the

P-and C-discriminants of the equation $8p^3x = y\left(12p^2 - 9\right)$ **(20)**

- Solve

$(i)\ \dfrac{d^2y}{dx^2} + \dfrac{2}{x}\dfrac{dy}{dx} + \dfrac{a^2}{x^4}y = 0$

$(ii)\ \dfrac{d^2y}{dx^2} + \left(\tan x - 3\cos x\right)\dfrac{dy}{dx} + 2y\cos^2 x = \cos^4 x.$

by varying parameters. **(20)**

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