SECTION A

Q1. (a) Let R be the ring of $n \times n$ matrices over reals. Show that R has only two ideals namely $\{0\}$ and R. 8

(b) Show that the series $\frac{1}{(1+a)^p} - \frac{1}{(2+a)^p} + \frac{1}{(3+a)^p} - \dots, a > 0$ is

(i) absolutely convergent if $p > 1$.

(ii) conditionally convergent if $0 < p \le 1$. 8

(c) If $f'(x) = (x-a)^{2n} (x-b)^{2m+1}$, where m, n are positive integers, show that f has neither a maximum nor a minimum at a and f has a local minimum at b. 8

(d) Let $f(z) = u(r, \theta) + iv(r, \theta)$ be an analytic function. If $u = -r^3 \sin 3\theta$, then construct the corresponding analytic function $f(z)$ in terms of z. 8

(e) Find all optimal solutions of the following linear programming problems graphically :

(i) Maximize

$$z = 3x_1 + 6x_2$$

subject to

$$x_1 + x_2 \le 8$$
$$x_1 - x_2 \le 4$$
$$2x_1 - x_2 \ge 4$$
$$x_1, x_2 \ge 0$$

5

(ii) The LPP in part (i) with the first constraint $x_1 + x_2 \le 8$ changed to $x_1 + 2x_2 \le 12$. 3

Q2. (a) (i) If G be a group of even order, then show that there exists an element 'a' other than the identity element, such that $a^2 = e$. 5

(ii) Prove that an ideal S of the ring Z of all integers is a maximal ideal, if S is generated by some prime integer. 5

(b) Examine the convergence of the improper integral $\int_0^\infty \frac{x^{p-1}}{1+x}\,dx$ and hence evaluate it. 15

(c) Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z-2)}$ and the residue at each pole and hence evaluate $\oint_C f(z)dz$ where C is the circle $|z| = 2\cdot5$. 15

Q3. (a) Evaluate $\lim\limits_{x \to 0} \frac{\int_0^{x^2} e^{\sqrt{1+t}}\,dt}{x^2}$ 10

(b) Prove that in a Unique Factorization Domain R, an element is prime if and only if it is irreducible. 15

(c) Solve the LPP :

Maximize

$$z = 2x_1 + x_2 + x_3$$

subject to

$$x_1 + 2x_2 - x_3 \leq 3$$

$$x_1 - 2x_2 - 5x_3 \geq -9$$

$$x_1, x_2, x_3 \geq 0$$

by the simplex method. Write its dual problem and from the optimal table of the given problem, obtain the optimal solution of the dual problem. 15

Q4. (a) Let G and H be finite groups, such that

$$\gcd(|G|, |H|) = 1$$

Show that the trivial homomorphism is the only homomorphism from G into H. 10

(b) (i) Find the image of $|z - 3i| = 3$ under the mapping $w = \frac{1}{z}$. 8

(ii) Find the value of the integral

$$\int_0^{1+i} (x - y + ix^2)\, dz$$

along the straight line from $z = 0$ to $z = 1 + i$. 7

(c) Find the initial basic feasible solution of the following minimum cost transportation problem by the Vogel's Approximation Method (VAM). Using it, find the optimal solution and the minimum transportation cost. Is the optimal solution unique ? If not, find an alternative optimal solution. 15

		Destination				
		D_1	D_2	D_3	D_4	Availability
	O_1	3	5	8	2	50
Origin	O_2	5	7	2	9	40
	O_3	7	1	3	4	30
Demand		40	35	25	20	

SECTION B

Q5. (a) Obtain the partial differential equation by eliminating the arbitrary function f from the equation $f(x + y + z,\ x^2 + y^2 + z^2) = 0$. *8*

(b) Obtain the following approximate quadrature formula :

$$\int_0^3 f(x)\,dx = \frac{3}{8}\,[f(0) + 3f(1) + 3f(2) + f(3)]$$ *8*

(c) (i) Convert $(523.0234375)_{10}$ into an equivalent octal number and then convert it to its binary form.

(ii) If $x = (1D2.2)_{16}$ and $y = (52E.32)_{16}$, then find the value of $x + y$ in decimal system. *4+4*

(d) The velocity components in an unsteady three dimensional flow are given by $u = \frac{x}{1+t}$, $v = \frac{y}{1+t}$, $w = \frac{z}{1+t}$. Describe the streamlines and pathlines. *8*

(e) Is a system of two particles which are connected by a rod of constant length holonomic ? Justify your answer. *8*

Q6. (a) Using Charpit's method, find the complete integral of $yq + 3xp = 2(z - y^2p^2)$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. *10*

(b) Write down the algorithm for solving the differential equation $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$, numerically by Euler's method with step length h up to $x = x_n = x_0 + nh$.

Solve the following differential equation for $x = 1$ with step length $h = 0.2$ by using Euler's method :

$$\frac{dy}{dx} = x^2 + y,\ y(0) = 1.$$ *6+9*

(c) Derive the Hamilton equations for holonomic systems and use them to discuss the motion of a simple pendulum. *15*

Q7. (a) Show that the iteration formula for the Newton-Raphson method for finding the K^{th} root of a positive real number a is :

$$x_{n+1} = \frac{1}{K}\left[(K-1)x_n + \frac{a}{x_n^{K-1}}\right], \text{ where } K > 0.$$

Use this formula to find $\sqrt[3]{13}$, correct up to three decimal places. *3+7*

(b) Find the general solution of the partial differential equation

$[D^2 - (D')^2 - 3D + 3D']z = (1-x)(1-y) + e^{x+2y}$,

where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$. *15*

(c) Consider an inviscid incompressible fluid flow with velocity

$\bar{q} = \left(x, \frac{y}{1+t}, \frac{z}{2+t}\right)$ under the body force $\bar{F} = -gz\hat{k}$, where g is the gravitational constant. Find the pressure at a point (x, y, z) if $p(0, 0, 0) = p_0$. *15*

Q8. (a) Consider a source and a sink of equal strength at points $\left(\pm\frac{1}{4}a, 0\right)$ within a fixed circular boundary $x^2 + y^2 = a^2$. Determine the equation of streamlines. *10*

(b) Find the solution of the heat equation

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}, \; 0 < x < \pi, \; t > 0$$

under the boundary conditions $u(0, t) = 0 = u(\pi, t)$ and the initial condition

$$u(x, 0) = \begin{cases} x, & 0 \le x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi. \end{cases}$$ *15*

(c) Use Gauss-Jordan elimination method to solve the following system of equations :

$$3x_1 + x_2 + x_3 = 7$$

$$2x_1 + x_2 + 5x_3 = 13$$

$$x_1 + 4x_2 + x_3 = 9 \cdot 4$$

correct up to 2-significant figures. *15*