

MATHEMATICS

Paper – II

Time Allowed : **Three Hours**

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

*Answers must be written in **ENGLISH** only.*

SECTION A

- Q1. (a)** Let G be a finite commutative group. Let $n \in \mathbb{Z}$ be such that n and order of G are relatively prime. Show that the function $\phi : G \rightarrow G$ defined by $\phi(a) = a^n$, for all $a \in G$, is an isomorphism of G onto G . 8

- (b) Apply Cauchy's Principle of Convergence to prove that the sequence $\langle f_n \rangle$ defined by

$$f_n = 1 + \frac{1}{4} + \frac{1}{7} + \dots + \frac{1}{3n-2}$$

is not convergent. 8

- (c) Find $\frac{dy}{dx}$, when

$$f(x, y) = \log(x^2 + y^2) + \tan^{-1}\left(\frac{y}{x}\right) = 0,$$

on using derivatives of Implicit Functions. 8

- (d) An automobile dealer wishes to put four repairmen R_1, R_2, R_3 and R_4 to four different jobs J_1, J_2, J_3 and J_4 . But R_3 cannot do the job J_2 . The dealer has estimated the number of man-hours that would be required for each job-man on one-one basis as given in the following table :

	R_1	R_2	R_3	R_4
J_1	6	2	3	4
J_2	9	7	–	5
J_3	6	4	7	5
J_4	6	8	8	9

Formulate the above as a Linear Programming Problem. 8

- (e) If $f(z) = u + iv$ is any analytic function of the complex variable z and $u - v = e^x (\cos y - \sin y)$, find $f(z)$ in terms of z . 8

Q2. (a) Prove that every group is isomorphic to a permutation group. 10

(b) Examine the convergence of $\int_0^{\infty} \frac{dx}{(1+x)\sqrt{x}}$ and find its value, if possible. 15

(c) Find the Taylor's series expansion of a function of the complex variable

$$f(z) = \frac{1}{(z-1)(z-3)}$$

about the point $z = 4$. Find its radius of convergence. 15

Q3. (a) Examine the existence of maxima and minima of the function,
 $u(x, y) = xy + \frac{8}{x} + \frac{8}{y}$. 10

(b) (i) Let R be a non-zero commutative ring with unity. If every ideal of R is prime, prove that R is a field.

(ii) Let R be a commutative ring with unity such that $a^2 = a, \forall a \in R$.
If I be any prime ideal of R , find all the elements of $\frac{R}{I}$. 8+7

(c) Consider the following Linear Programming Problem as primal :

Minimize $z = 30x_1 + 20x_2$

s/t, $3x_1 + 5x_2 \geq 100$

$$2x_1 + x_2 \geq 120$$

$$5x_1 + 3x_2 \geq 90$$

$$x_1, x_2 \geq 0$$

Then using the principle of duality, find the optimal solution of the primal. 15

Q4. (a) Show that

$$\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4 \cos \theta} d\theta = \frac{\pi}{12}. \quad 15$$

(b) Show that an element x in a Euclidean domain is a unit if and only if $d(x) = d(1)$, where the notations have their usual meanings. 10

(c) Starting with Least Cost Method, find all the solutions to the following transportation problem : 15

		Warehouses				
		I	II	III	IV	
Plants	A	8	6	5	3	18
	B	6	7	6	8	20
	C	10	8	4	5	18
		15	16	12	13	
		Demand				

SECTION B

Q5. (a) Find the complete primitive of

$$4r - 4s + t = 16 \log_e (x + 2y),$$

r, s, t bear their usual meanings.

8

(b) From the following table, estimate the number of students who obtained marks between 40 and 46 :

8

Marks :	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students :	32	43	55	40	30

(c) Consider the following integers and their 8-bits binary representations :

$$13 = 00001101, \quad 20 = 00010100$$

Perform the following bitwise operations and express the results in decimal system :

2+2+2+2

(i) $13 \& 20$ (Bitwise AND)

(ii) $13 | 20$ (Bitwise OR)

(iii) $13 \wedge 20$ (Bitwise XOR)

(iv) ~ 20 (Bitwise Compliment)

(d) Examine the motion of a particle sliding on a parabolic wire given by $x^2 = 2y$.

8

(e) Find the orthogonal trajectory of the following family of curves :

$$x^2 - y^2 = a^2$$

Then sketch the two families to demonstrate whether they cut orthogonally.

8

Q6. (a) Solve the following by Charpit's method : 10

$$pxy + pq + qy = yz, \quad p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}$$

(b) Using Regula-Falsi method, find the fourth root of 28 correct to three decimal places. 15

(c) Verify whether the motion given by

$$\vec{q} = (3x \hat{i} - 2y \hat{j}) xy^2$$

is a possible fluid motion. If so, is it of the potential kind ? Accordingly find out the streamlines and the velocity potential or the angular velocity if the fluid was replaced by a rigid solid. 15

Q7. (a) Write down the algorithm and flowchart of Runge-Kutta method of fourth order to find the numerical solution at $x = 0.8$ for

$$\frac{dy}{dx} = \sqrt{2(x+y)}, \quad y(0.4) = 0.82. \quad \text{7+8}$$

Assume the step length $h = 0.2$.

(b) Discuss the flow given by the complex potential

$$w = \log_e \left(z - \frac{a^2}{z} \right).$$

Draw sketches of the streamlines and explain the flow directions along the streamlines. 15

(c) Solve the following differential equation : 10

$$(y^2 + z^2 - x^2) p - 2xyq + 2xz = 0, \quad p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}$$

Q8. (a) Derive the Lagrange's equation for a spherical problem. 15

(b) Solve the following system of equations by Gauss-Seidel method : 10

$$20x + y - 3z = 16$$

$$2x + 20y - z = -19$$

$$3x - 2y + 20z = 25$$

starting with the initial solution $x_0 = y_0 = z_0 = 0$.

(c) Find the singular solution of $yp^2 - 2xp + y = 0$. Also trace the graph. 15