

INSTITUTE FOR IAS/IFoS/CSIR/GATE EXAMINATIONS

# MATHEMATICS

by

## K. VENKANNA

(14 years teaching experience)

### BATCH BEGINS

- CSIR - December 2013 / June 2014
- GATE 2014
- GATE 2014 Engineering Mathematics Module Course

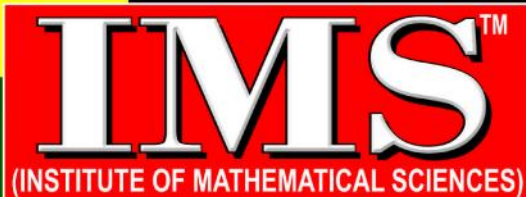
From  
**11<sup>th</sup>**  
JULY 2013

*Admission* **OPEN**

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## *About the*IMS

With the emerging opportunities in the field of Science and Tech, the importance of mathematics has become indispensable. Now-a-days mathematics is in use some way or the other in almost all dimensions of the human creativity. The increasing popularity of this subject and its application has been impacting the entire system in manifold ways. In spite of all these positive aspects, there is one aspect of this subject as well that has been hampering the entire chunk of students badly. That aspect is due to the lack of guidance for which the most of the aspirants are the worst sufferers.

After spending many years mentoring the students of IAS aspirants for mathematics optional, I realized that a huge sections of students who are preparing for other competitive examinations like CSIR-NET, GATE and many more are deprived of my guidance and they are the victims of low-standard institutes. These institutes being not at par give guidance superficially and instead of catering the actual need and the requirement of the examination, they are making it, the years plan resulting in the great loss of precious time and money of the aspirants. IMS institute takes the initiative to cater the needs of the aspirants appearing in CSIR-NET and GATE examinations. Our multi-dimensional approaches do help the students to click their goals in just one shot.

Students are cordially invited to join this course and can see the difference, this difference is not only in teaching style, but also in study materials too. IMS guarantees success with the difference.

**“We aim to train rather than just educate”**

**It is not one of the Institutes.**

**It is the only Institute**

**Don't take a chance**

**Go for a thought.....IMS (Institute of Mathematical Sciences)**

Thanks,

Wish You All the Best

**K. VENKANNA**  
**(DIRECTOR)**  
**IMS**

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**Council of Scientific & Industrial Research**  
**Human Resource Development Group**  
**CSIR Complex, New Delhi - 110012**

No.6/ Fellowship (ENH) 2k10/ EMR - 1

26th November, 2010

**OFFICE MEMORANDUM**

**SUB-REVISION OF FELLOWSHIPS/ ASSOCIATESHIPS**

The Government Body (GB), CSIR in its 176th meeting held on June 2010 considered the proposal for enhancement of various Fellowships. The enhanced rate of Fellowships, effective from 1st April 2010, are as follows:

S.No.	Research Fellows/Research Associates	Existing emoluments (Rs.)		Revised emoluments (Rs.)	
		1st two years	After 2nd years	1st two years	After 2nd years
1.	JRF - NET	12000	14000	16000	18000
2.	JRF-NET-LS or GATE qualified in CSIR supported EMR projects	12000	14000	12000	14000
3.	JRF-GATE with degree in professional courses (tenable at CSIR labs)	12000	14000	16000	18000
4.	SRF-Direct with Graduate degree in professional courses	14000	14000	18000	18000
5.	SRF-Direct with postgraduate degree in professional courses	14000	15000	18000	18000
6.	Shyama Prasad Mukherjee Fellowship (SPMF)	14000	15000	18000	20000
7.	SRF-Extended (one year only)	15000	18000	20000	24000
8.	SRF - Extended (one year only)	15000	-	20000	-
9.	Research Associate - I	16000	16000	22000	22000
10.	Research Associate - II	17000	17000	23000	23000
11.	Research Associate - III	18000	18000	24000	24000
12.	CSIR Diamond Jubilee Research Internship *(tendable at CSIR labs only)	11500	-	15000	-

Contingency amount @ Rs. 70000/- p.a. in case of SPM Fellows & @ Rs. 200000/- p.a. in case of all other Research Fellows/ Associates remains the same.

**\* Payment will be made from Laboratory Reserve Fund (LRF) of the laboratory.**

**[O. Omman Panicker]**  
**Deputy Secretary (EMR & Cx.)**

## About the CSIR-UGC (NET)

CSIR-NET Exam is mandatory for candidates aspiring to teach in various degree colleges/ universities in all over India. **CSIR-UGC conducts JRF/NET exam twice a year in the month of June and December. The exam will be conducted in different subjects like Mathematical Sciences, Physical Sciences & Life Sciences etc.**

### *Recent changes in CSIR-UGC:*

- ❖ Observing that “the courts should not venture into academic field, Delhi High Court has upheld the mandatory requirement of clearing the NET or SLET for appointment to the post of Lecturer.
- ❖ The University Grants Commission (UGC) framed the Rule & Regulations – 2009 in July. Which says that NET or SLET is mandatory for appointment of Lecturers.
- ❖ Atleast 55% marks or equivalent grade is required in master degree for NET qualification.
- ❖ Atleast one Professor in each Dept. in P.G. College is a requirement.
- ❖ The new regulations have also created an additional post senior professor. Accordingly, the new hierarchy in ascending order is assistant professor, associate professor, professor and senior professor.
- ❖ One post of a professor in each department of the postgraduate college, and of 10% posts in an undergraduate college shall be of those from professors only.

## Conditions of Eligibility for CSIR-JRF(NET)

### *Educational Qualification*

BS-4 years program/BE/B. Tech/B. Pharma/MBBS/Integrated BS-MS/M.Sc. or Equivalent degree with at least 55% marks for General & OBC (50% for SC/ST candidates, Physically and Visually handicapped candidates) Candidate enrolled for M.Sc. or having completed 10+2+3 years of the above qualifying examination are also eligible to apply in the above subject under the Result Awaited (RA) category on the condition that they complete the qualifying degree with requisite percentage of marks within the validity period of two years to avail the fellowship from the effective date of award.

Such candidates will have to submit the attestation format (Given at the reverse of the application form) duly certified by the Head of the Department/Institute from where the candidate is appearing or has appeared.

B.Sc. (Hons) or equivalent degree holders or students enrolled in integrated MS-Ph.D program with at least 55% marks for General & OBC candidates; 50% for SC/ST candidates, Physically and Visually handicapped candidates are also eligible to apply.

Candidates with bachelor's degree, whether Science, engineering or any other discipline, will be eligible for fellowship only after getting registered/enrolled for Ph.D/integrated Ph.D. program within the validity period of two years.

The eligible for lectureship of NET qualified candidates will be subject to fulfilling the criteria laid down by UGC. Ph.D. degree holders who have passed Master's degree prior to 19th September, 1991 with at least 50% marks are eligible to apply for Lectureship only.

### *Age Limit & Relaxation:*

**The age limit for admission to the Test is as under:**

For JRF (NET): Maximum 28 years (upper age limit may be relaxed up to 5 years in case of candidates belonging to SC/ST/OBC (As per GOI central list), Physically handicapped/Visually handicapped and female applicants).

**For Lectureship (NET): No upper age limit.**

## About the GATE Examination

Graduate Aptitude Test in Engineering (GATE) is an all India examination that primarily tests a comprehensive understanding of various undergraduate subjects in Engineering and Technology. The GATE score of a candidate reflects a relative performance level in a particular paper in the exam across several years. The score is used for admissions to post-graduate engineering programmes (eg. M.E., M.Tech, direct Ph.D.) in Indian higher education institutes with financial assistance provided by MHRD and other Government agencies. The score may also be used by Public sector units for employment screening purposes.

**NOTE:**

- The GATE score card is valid for the two years.
- There is no limit for number of attempts and age for the GATE examination, so candidate can appear in the GATE examination at any age and many times as per candidate wish.
- GATE Score:** After the evaluation of the answers, the raw marks obtained by a candidate will be converted to a normalized GATE Score.  
From 2013, the GATE score will be computed by a new formula.  
The GATE Score of a candidate is computed from:

$$S = S_q + (S_t - S_q) \frac{M - M_q}{\overline{M}_t - M_q},$$

where,

$S$  = GATE Score (normalised) of a candidate,

$M$  = Marks obtained by a candidate in a paper,

$M_q$  = Qualifying Marks for general category candidates in the paper,

$\overline{M}_t$  = Average Marks for top 0.1% or 10 (which ever is higher) of candidates in the paper,

$S_t$  = GATE Score assigned to  $\overline{M}_t$  (around 900), and

$S_q$  = GATE Score assigned to  $M_q$  (around 300),

$M_q$  is usually 25 marks (out of 100) or  $\sim + \dagger$ , which ever is higher. Here  $\sim$  is the mean of marks in a paper and  $\dagger$  is the standard deviation.

- Generally, GATE results will be announced in every around march 15.

**Eligibility for GATE according to the 2013 notification**

Only the following categories of candidates are eligible to appear for GATE 2013. Necessary supporting documents must be submitted ONLINE or by post during the submission of the application form for the exam. Please read this carefully and make sure that your year of qualification is not later than what is specified below.

Qualifying Degree (Short)	Qualifying Degree/ Examination (Descriptive)	Description of Eligible Candidates	Year of qualification cannot be later than	Copies of Certificates to be submitted	
				Passed in the year 2012 or earlier	Expected to complete in 2013 or later
B.E./B.Tech/B.Arch	Bachelor's degree in Engineering/Technology/Architecture (4 years after 10+2/ Post B.Sc./ Post-Diploma)	4 <sup>th</sup> year of completed	2013	Degree Certificate/ Provisional Certificate/ Course Completion Certificate	Certificate from Principal
M.Sc/M.A./MCA equivalent	Master's degree in any branch of Science/ Mathematics/ Statistics/ Computer Applications or equivalent	Final year or completed	2013	Degree Certificate/ Provisional Certificate/ Course Completion Certificate (pertaining to Masters degree)	Certificate from Principal
Int. M.E./M.Tech or DD (after 10+2 or Diploma)	Integrated Master's degree programs or Dual Degree programs in Engineering/ Technology (Five years programme)	4 <sup>th</sup> /5 <sup>th</sup> year or completed	2014	Degree Certificate/ Provisional Certificate/ Course Completion Certificate	Certificate from Principal
Int. M.E/M.Tech (Post BSc)	Post-BSc Integrated Master's degree programs in Engineering/ Technology (Four year programme)	2 <sup>nd</sup> /3 <sup>rd</sup> /4 <sup>th</sup> year or completed	2015	Degree Certificate/ Provisional Certificate/ Course Completion Certificate	Certificate from Principal
Professional Society Examinations (equivalent to B.E./B.Tech/B.Arch)	B.E./B.Tech equivalent examinations, of Professional Societies, recognized by MHRD/U.PSC/ AICTE (e.g. AMIE by Institution of Engineers-India, AMICE by the Institute of Civil Engineers India)	Completed section A or equivalent of such professional courses	NA	Professional Certificate/ Provisional Certificate/ Course Completion/ Membership Certificate issued by the Society or Institute	Copy of Marksheet from Section "A"

### ***Certificate from Principal***

Candidates who have to submit a certificate from their Principal, as determined from the above table, have to obtain a signature from their principal on a certificate that will be printed on the application PDF file provided after completion of online application submission.

### ***Candidates with backlogs***

Candidates who have appeared in the final semester/year exam in 2012, but with a backlog (arrears/failed subjects) in any of the papers in their qualifying degree should submit

1. A copy of any one of the marks sheets of the final year, OR
2. A letter from the principal indicating that the student has a backlog from an earlier semester/year to be cleared, and therefore cannot produce a course completion certificate now. This certificate will also be present in the last portion of the PDF application form provided to you after you submit application online.

### **GATE Papers according to the 2013 notification**

GATE 2013 will be conducted in the following subjects (also referred to as “papers”). Candidates must familiarize with the paper code for the paper of their choice, as this knowledge will be required at the time of application form submission and appearing for the examination.

GATE PAPER	CODE	GATE PAPER	CODE
Aerospace Engineering	AE	Instrumentation Engineering	IN
Agricultural Engineering	AG	Mathematics	MA
Architecture and Planning	AR	Mechanical Engineering	ME
Biotechnology	BT	Mining Engineering	MN
Civil Engineering	CE	Metallurgical Engineering	MT
Chemical Engineering	CH	Physics	PH
Computer Science and Information Technology	CS	Production and Industrial Engineering	PI
Chemistry	CY	Textile Engineering and Fibre Science	TF
Electronics and Communication Engineering	EC	Engineering Sciences	XE*
Electrical Engineering	EE	Life Sciences	XL**
Geology and Geophysics	GG		
*XE PAPER SECTIONS	CODE	*XE PAPER SECTIONS	CODE
Engineering Mathematics (Compulsory)	A	Chemistry (Compulsory)	
Fluid Mechanics	B	Biochemistry	
Materials Science	C	Botany	
Solid Mechanics	D	Microbiology	
Thermodynamics	E	Zoology	
Polymer Science and Engineering	F	Food Technology	
Food Technology	G		

### ***GATE Examination Schedule according to 2013 notification***

GATE Paper Codes	Examination Time	Examination Type
AR, CE, GG, MA, MT, PH, and TF	09:00 hrs – 12:00 hrs	ONLINE
AE, AG, BT, CH, CY, MN, XE and XL	14:00 hrs – 17:00 hrs	ONLINE
CS, ME and PI	09:00 hrs – 12:00 hrs	OFFLINE
EC, EE and IN	14:00 hrs – 17:00 hrs	OFFLINE

**ONLINE Examination:** A computer based test (CBT) where the candidate will use a computer mouse to choose a correct answer or enter a numerical answer via a virtual keypad.

**OFFLINE Examination:** A paper based examination where the candidate will mark the correct answer out of four options in an Optical Response Sheet (ORS) by darkening the appropriate bubble with a pen.



## CSIR New Pattern of Examination

(Council of Scientific and Industrial Research Human Resources Development Group Examination Unit)

### CSIR-UGC (NET) Exam for Award of Junior Research Fellowship and Eligibility for Lectureship

## MATHEMATICAL SCIENCES

### Exam Scheme

**Time: 3 Hours**

**Maximum Marks:200**

From June, 2011 CSIR-UGC (NET) Exam for Award of Junior Research Fellowship and Eligibility for Lectureship shall be a Single Paper Test having Multiple Choice Question (MCQs). The question paper shall be divided in three parts.

#### **PART-A**

This part shall carry 20 questions pertaining to General Aptitude with emphasis on logical reasoning, graphical analysis, analytical and numerical ability, quantitative comparison, series formation, puzzle etc. The candidates shall be required to answer any 15 questions. Each question shall be of two marks. The total marks allocated to this section shall be 30 out of 200.

#### **PART-B**

This part shall contain 40 Multiple Choice Questions (MCQs) generally covering the topics given in the syllabus. A candidate shall be required to answer any 25 questions. Each question shall be of three marks. The total marks allocated to this section shall be 75 out of 200.

#### **PART-C**

This part shall contain 60 questions that are designed to test a candidate's knowledge of scientific concepts and or application of the scientific concepts. The questions shall be of analytical nature where a candidate is expected to apply the scientific knowledge to arrive at the solution to the given scientific problem. The questions in this part shall have multiple correct options. Credit in a question shall be given only on identification of all the correct options. No credit shall be allowed in a question if any incorrect option is marked as correct answer. No partial credit is allowed. A candidate shall be required to answer any 20 questions. Each question shall be of 4.75 marks. The total marks allocated to this section shall be 95 out of 200.

- ❖ For Part 'A' and 'B' there will be Negative marking @25% for each wrong answer. No Negative marking for Part 'C'.
- ❖ To enable the candidates to go through the questions, the question paper booklet shall be distributed 15 minutes before the scheduled time of the exam.
- ❖ On completion of the exam i.e. at the scheduled closing time of the exam, the candidates shall be allowed to carry the question Paper Booklet. No candidate is allowed to carry the Question Paper Booklet in case he/she chooses to leave the test before the scheduled closing time.

**Source: CSIR**

### (Common Syllabus for Part 'B & C')

#### UNIT-1

#### **Analysis:**

- ❖ Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf.
- ❖ Bolzano Weierstrass theorem, Heine Borel theorem.
- ❖ Continuity, uniform continuity, differentiability, mean value theorem.

- ❖ Sequences and series of functions, uniform convergence.
- ❖ Riemann sums and Riemann integral, Improper Integrals.
- ❖ Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral.
- ❖ Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems.
- ❖ Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

### **Linear Algebra:**

- ❖ Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations.
- ❖ Algebra of matrices, rank and determinant of matrices, linear equations.
- ❖ Eigenvalues and eigenvectors, Cayley-Hamilton theorem.
- ❖ Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms.
- ❖ Inner product spaces, orthonormal basis.
- ❖ Quadratic forms, reduction and classification of quadratic forms .

## **UNIT – 2**

### **Complex Analysis:**

- ❖ Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions.
- ❖ Analytic functions, Cauchy-Riemann equations.
- ❖ Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem.
- ❖ Taylor series, Laurent series, calculus of residues.
- ❖ Conformal mappings, Mobius transformations.

### **Algebra:**

- ❖ Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements.
- ❖ Fundamental theorem of arithmetic, divisibility in  $\mathbb{Z}$ , congruences, Chinese Remainder Theorem, Euler's  $\phi$ - function, primitive roots.
- ❖ Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems.
- ❖ Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain.
- ❖ Polynomial rings and irreducibility criteria.
- ❖ Fields, finite fields, field extensions, Galois Theory.

### **Topology:**

- ❖ basis, dense sets, subspace and product topology, separation axioms, connectedness and compactness.

## **UNIT – 3**

### **Ordinary Differential Equations (ODEs):**

- ❖ Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs.
- ❖ General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

### **Partial Differential Equations (PDEs):**

- ❖ Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs.



- ❖ Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

### **Numerical Analysis:**

- ❖ Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

### **Calculus of Variations:**

- ❖ Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema.
- ❖ Variational methods for boundary value problems in ordinary and partial differential equations.

### **Linear Integral Equations:**

- ❖ Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels.
- ❖ Characteristic numbers and eigenfunctions, resolvent kernel.

### **Classical Mechanics:**

- ❖ Generalized coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action, Two-dimensional motion of rigid bodies, Euler's dynamical equations for the motion of a rigid body about an axis, theory of small oscillations.

## **UNIT – 4**

- ❖ Descriptive statistics, exploratory data analysis Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (i.i.d. case).
- ❖ Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson and birth-and-death processes.
- ❖ Standard discrete and continuous univariate distributions. sampling distributions, standard errors and asymptotic distributions, distribution of order statistics and range.
- ❖ Methods of estimation, properties of estimators, confidence intervals.
- ❖ Tests of hypotheses: most powerful and uniformly most powerful tests, likelihood ratio tests. Analysis of discrete data and chi-square test of goodness of fit. Large sample tests.
- ❖ Simple nonparametric tests for one and two sample problems, rank correlation and test for independence. Elementary Bayesian inference.
- ❖ Gauss-Markov models, estimability of parameters, best linear unbiased estimators, confidence intervals, tests for linear hypotheses. Analysis of variance and covariance. Fixed, random and mixed effects models. Simple and multiple linear regression.
- ❖ Elementary regression diagnostics. Logistic regression.
- ❖ Multivariate normal distribution, Wishart distribution and their properties. Distribution of quadratic forms. Inference for parameters, partial and multiple correlation coefficients and related tests. Data reduction techniques: Principle component analysis, Discriminant analysis, Cluster analysis, Canonical correlation.
- ❖ Simple random sampling, stratified sampling and systematic sampling. Probability proportional to size sampling. Ratio and regression methods.
- ❖ Completely randomized designs, randomized block designs and Latin-square designs. Connectedness and orthogonality of block designs, BIBD. 2K factorial experiments: confounding and construction.

- ❖ Hazard function and failure rates, censoring and life testing, series and parallel systems.
- ❖ Linear programming problem, simplex methods, duality. Elementary queuing and inventory models. Steady-state solutions of Markovian queuing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1.
- ❖ All students are expected to answer questions from Unit-1. Mathematics students are expected to answer additional questions from Unit-II and III. Statistics students are expected to answer additional questions from Unit-IV.

## **GATE: Examination Related Information**

### *Pattern of Question Papers and Marking Scheme*

#### **Duration and Exam Type:**

The GATE examination consists of a single paper of 3 hours duration that contains 65 questions carrying a maximum of 100 marks. The question paper will consist of only objective questions.

#### **Pattern of Question Papers:**

The examination for the papers with codes AE, AG, AR, BT, CE, CH, CY, GG, MA, MN, MT, PH, TF, XE and XL will be conducted ONLINE using computers where the candidates will be required to select the answer for each question using a mouse. For all other papers (CS, EC, EE, IN, ME & PI), the candidates will have to mark the correct choice on an Optical Response Sheet (ORS) by darkening the appropriate bubble against each question.

In all the papers, there will be a total of 65 questions carrying 100 marks, out of which 10 questions carrying total of 15 marks are in General Aptitude (GA). The remaining 85 % of the total marks is devoted to the syllabus of the paper (as indicated in the syllabus section).

#### **GATE 2013 would contain questions of four different types in various papers:**

- (i) **Multiple choice questions** carrying 1 or 2 marks each; Each of the multiple choice objective questions in all papers and sections will contain four answers, of which one correct answer is to be marked.
- (ii) **Common data questions** (which are also multiple choice questions), where two successive questions use the same set of input data;
- (iii) **Linked answer questions** (which are also multiple choice questions), where the answer to the first question in the pair is required to answer its successor;
- (iv) **Numerical answer questions**, where the answer is a number, to be entered by the candidate.

#### **Marking Scheme**

For 1 mark multiple-choice questions, 1/3 mark will be deducted for a wrong answer. Likewise, for 2 marks multiple-choice questions, 2/3 mark will be deducted for a wrong answer. However, for the linked answer question pair, where each question carries 2 marks, 2/3 mark will be deducted for a wrong answer to the first question only. There is no negative marking for wrong answer to the second question of the linked answer question pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated. There is no negative marking for numerical answer type questions.

#### **General Aptitude (GA) Questions**

In all papers, GA questions are of multiple choice type, and carry a total of 15 marks. The GA section includes 5 questions carrying 1 mark each (sub-total 5 marks) and 5 questions carrying 2 marks each (sub-total 10 marks).

#### **Question papers other than GG, XE and XL**

These papers would contain 25 questions carrying one mark each (sub-total 25 marks) and 30 questions carrying two marks each (sub-total 60 marks). Out of these, two pairs of questions would be common data questions, and two pairs of questions would be linked answer questions.

In the ONLINE papers, the question paper will consist of questions of multiple choice type and numerical answer type. For multiple choice type questions, each question will have four choices for the answer. For numerical answer type questions, each question will have a number as the answer and choices will not be given. Candidates will have to enter the answer using a virtual keypad.

# GATE Syllabus

## General Aptitude (GA): Common to All Papers

### Engineering

1. **Verbal Ability:** English grammar, sentence completion, verbal analogies, word groups, instructions, critical reasoning and verbal deduction.
2. **Numerical Ability:** Numerical computation, numerical estimation, numerical reasoning and data interpretation.

## Syllabus Mathematics

### Linear Algebra:

Finite dimensional vector spaces; Linear transformations and their matrix representations, rank; systems of linear equations, eigen values and eigen vectors, minimal polynomial, Cayley-Hamilton Theorem, diagonalisation, Hermitian, Skew-Hermitian and unitary matrices; Finite dimensional inner product spaces, Gram-Schmidt orthonormalization process, self-adjoint operators.

### Complex Analysis:

Analytic functions, conformal mappings, bilinear transformations; complex integration: Cauchy's integral theorem and formula; Liouville's theorem, maximum modulus principle; Taylor and Laurent's series; residue theorem and applications for evaluating real integrals.

### Real Analysis:

Sequences and series of functions, uniform convergence, power series, Fourier series, functions of several variables, maxima, minima; Riemann integration, multiple integrals, line, surface and volume integrals, theorems of Green, Stokes and Gauss; metric spaces, completeness, Weierstrass approximation theorem, compactness; Lebesgue measure, measurable functions; Lebesgue integral, Fatou's lemma, dominated convergence theorem.

### Ordinary Differential Equations:

First order ordinary differential equations, existence and uniqueness theorems, systems of linear first order ordinary differential equations, linear ordinary differential equations of higher order with constant coefficients; linear second order ordinary differential equations with variable coefficients; method of Laplace transforms for solving ordinary differential equations, series solutions; Legendre and Bessel functions and their orthogonality.

### Algebra:

Normal subgroups and homomorphism theorems, automorphisms; Group actions, Sylow's theorems and their applications; Euclidean domains, Principal ideal domains and unique factorization domains. Prime ideals and maximal ideals in commutative rings; Fields, finite fields.

### Functional Analysis:

Banach spaces, Hahn-Banach extension theorem, open mapping and closed graph theorems, principle of uniform boundedness; Hilbert spaces, orthonormal bases, Riesz representation theorem, bounded linear operators.

### Numerical Analysis:

Numerical solution of algebraic and transcendental equations: bisection, secant method, Newton-Raphson method, fixed point iteration; interpolation: error of polynomial interpolation, Lagrange, Newton interpolations; numerical differentiation; numerical integration: Trapezoidal and Simpson rules, Gauss Legendre quadrature, method of undetermined parameters; least square polynomial approximation; numerical solution of systems of linear equations: direct methods (Gauss elimination, LU decomposition); iterative methods (Jacobi and Gauss-Seidel); matrix eigenvalue problems: power method, numerical solution of ordinary differential equations: initial value problems: Taylor series methods, Euler's method, Runge Kutta methods.

**Partial Differential Equations:**

Linear and quasilinear first order partial differential equations, method of characteristics; second order linear equations in two variables and their classification; Cauchy, Dirichlet and Neumann problems; solutions of Laplace, wave and diffusion equations in two variables; Fourier series and Fourier transform and Laplace transform methods of solutions for the above equations.

**Mechanics:**

Virtual work, Lagrange's equations for holonomic systems, Hamiltonian equations.

**Topology:**

Basic concepts of topology, product topology, connectedness, compactness, countability and separation axioms, Urysohn's Lemma.

**Probability and Statistics:**

Probability space, conditional probability, Bayes theorem, independence, Random variables, joint and conditional distributions, standard probability distributions and their properties, expectation, conditional expectation, moments; Weak and strong law of large numbers, central limit theorem; Sampling distributions, UMVU estimators, maximum likelihood estimators, Testing of hypotheses, standard parametric tests based on normal,  $X^2$ ,  $t$ ,  $F$ -distributions; Linear regression; Interval estimation.

**Linear programming:**

Linear programming problem and its formulation, convex sets and their properties, graphical method, basic feasible solution, simplex method, big-M and two phase methods; infeasible and unbounded LPP's, alternate optima; Dual problem and duality theorems, dual simplex method and its application in post optimality analysis; Balanced and unbalanced transportation problems,  $u$ - $v$  method for solving transportation problems; Hungarian method for solving assignment problems.

**Calculus of Variation and Integral Equations:**

Variation problems with fixed boundaries; sufficient conditions for extremum, linear integral equations of Fredholm and Volterra type, their iterative solutions.

**SAMPLE QUESTIONS**

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|---|---|
| <p>1. Let <math>w_1, w_2, w_3</math> be subspaces of a vector space <math>V</math> such that <math>w_3 \subset w_1</math>. Then which one of the following is correct ?</p> <p>(a) <math>w_1 \cap (w_2 + w_3) = w_2 + w_1 \cap w_3</math></p> <p>(b) <math>w_1 \cap (w_2 + w_3) = w_3 + (w_1 \cap w_2)</math></p> <p>(c) <math>w_1 \cup (w_2 + w_3) = w_2 + w_1 \cup w_3</math></p> <p>(d) None of the above</p> <p>2. Let <math>\alpha_1, \alpha_2</math> and <math>\alpha_3</math> be vectors of a vector space <math>V</math> over the field <math>F</math>. If <math>r</math> and <math>s</math> are arbitrary elements of <math>F</math> and the set <math>\{\alpha_1, \alpha_2, r\alpha_1 + s\alpha_2 + \alpha_3\}</math> is linearly dependent, then <math>\{\alpha_1, \alpha_2, \alpha_3\}</math> is:</p> <p>(a) linearly dependent set</p> <p>(b) a null set</p> <p>(c) linearly independent set</p> <p>(d) None of the above</p> <p>3. Consider the following vector spaces over the reals:</p> <p>1. The set of all complex numbers with</p> | <p>usual operations</p> <p>2. The set of all polynomials with real coefficients of degree <math>\leq 3</math></p> <p>3. The set of all <math>x = 2t, y = -t, z = 4t, t \in \mathbb{R}</math></p> <p>4. The set of all <math>3 \times 3</math> matrices having real entries with usual operations</p> <p>The correct sequence of these vector spaces in decreasing order of their dimensions is:</p> <p>(a) 1, 2, 3, 4      (b) 2, 1, 4, 3</p> <p>(c) 4, 2, 1, 3      (d) 4, 3, 2, 1</p> <p>4. Consider the following assertions:</p> <p>1. Rank <math>(ST) = \text{Rank } S = \text{Rank } T</math></p> <p>2. Rank <math>(ST) = \text{Rank } S</math>, if <math>T</math> is non singular</p> <p>3. Rank <math>(ST) = \text{Rank } T</math>, if <math>T</math> is non singular</p> <p>where <math>S, T : V \rightarrow V</math> are linear transformations of a finite dimensional vector space. Which of these is/are correct</p> <p>(a) 1 Only      (b) 2 Only</p> <p>(c) 1 &amp; 2      (d) 2 &amp; 3</p> |
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5. Consider the linear transformation  $T: \mathbb{R}^4 \rightarrow \mathbb{R}^4$  given by

$$T(x, y, z, u) = (x, y, 0, 0) \forall (x, y, z, u) \in \mathbb{R}^4$$

Then which one of the following is correct

- (a) Rank of T > Nullity of T  
 (b) Rank of T = Nullity of T = 3  
 (c) Nullity of T > Rank of T  
 (d) Rank of T = Nullity of T = 2
6. If V is the real vector space of all mappings from  $\mathbb{R}$  to  $\mathbb{R}$ ,  $V_1 = \{f \in V / f(-x) = f(x)\}$

and  $V_2 = \{f \in V / f(-x) = -f(x)\}$ , then which one of the following is correct

- (a) Neither  $V_1$  nor  $V_2$  is a subspace of V.  
 (b)  $V_1$  is a subspace of V, but  $V_2$  is not a subspace of V.  
 (c)  $V_1$  is not a subspace of V, but  $V_2$  is a subspace of V.  
 (d) Both  $V_1$ , and  $V_2$  are subspaces of V.
7. Consider the field  $\mathbb{R}$  of real numbers and the following spaces:

1. Set of real valued functions on  $[0, 1]$

having discontinuity at  $x = \frac{1}{2}$ .

2.  $\{(x, y) \in \mathbb{R}^2 / x > 0\}$

Which of the above is/are vector spaces(s)?

- (a) 1 Only (b) 2 Only  
 (c) Both 1 & 2 (d) Neither 1 nor 2
8. The determinant of the matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- (a) 0 (b) -9  
 (c) -27 (d) 1

9. The eigenvalues of the matrix  $\begin{pmatrix} 1 & 0 & 0 & -\frac{a}{2} \\ 0 & 1 & 0 & -\frac{a}{2} \\ 0 & 0 & 1 & -\frac{a}{2} \\ 0 & 0 & 0 & a \end{pmatrix}$  are

- (a) 1 and a (b) 1 and -a

- (c)  $\frac{1}{2}$  and  $-\frac{a}{2}$  (d)  $\frac{1}{2}$  and  $\frac{a}{2}$

10. Let  $M = \begin{pmatrix} 1 & 1+i & 2-i \\ 1-i & 2 & 3+i \\ 2+i & 3-i & 3 \end{pmatrix}$ . If

$$B = \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix},$$

where  $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ ,  $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$  and  $\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$  are linearly

independent eigenvectors of M, then the main diagonal of the matrix  $B^{-1}MB$  has

- (a) exactly one real entry  
 (b) exactly two real entries  
 (c) exactly three real entries  
 (d) no real entry
11. Let A be a  $3 \times 3$  matrix with eigenvalues 1, -1 and 3. Then
- (a)  $A^2 + A$  is non-singular  
 (b)  $A^2 - A$  is non-singular  
 (c)  $A^2 + 3A$  is non-singular  
 (d)  $A^2 - 3A$  is non-singular
12. A  $2 \times 2$  real matrix A is diagonalizable if and only if:
- (a)  $(\text{tr } A)^2 < 4\text{Det } A$   
 (b)  $(\text{tr } A)^2 > 4\text{Det } A$ .  
 (c)  $(\text{tr } A)^2 = 4\text{Det } A$ .  
 (d)  $\text{Tr } A = \text{Det } A$ .
13. The minimal polynomial of the  $3 \times 3$  real matrix

$$\begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix} \text{ is}$$

- (a)  $(X - a)(X - b)$  (b)  $(X - a)^2(X - b)$   
 (c)  $(X - a)^2(X - b)^2$  (d)  $(X - a)(X - b)^2$
14. Let V be the subspace of  $\mathbb{R}^3$  spanned by  $u = (1, 1, 1)$  and  $v = (1, 1, -1)$ . The orthonormal basis of V obtained by the Gram-Schmidt process on the ordered basis  $(u, v)$  of V is

- (a)  $\left\{ \left( \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right), \left( \frac{2}{3}, \frac{2}{3}, -\frac{4}{3} \right) \right\}$   
 (b)  $\{(1, 1, 0), (1, 0, 1)\}$

$$(c) \left\{ \left( \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right), \left( \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}} \right) \right\}$$

$$(d) \left\{ \left( \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right), \left( \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \right) \right\}$$

15. Let  $r = e^{2\pi i/5}$  and the matrix

$$M = \begin{bmatrix} 1 & r & r^2 & r^3 & r^4 \\ 0 & r & r^2 & r^3 & r^4 \\ 0 & 0 & r^2 & r^3 & r^4 \\ 0 & 0 & 0 & r^3 & r^4 \\ 0 & 0 & 0 & 0 & r^4 \end{bmatrix}$$

Then the trace of the matrix  $I + M + M^2$  is

- (a) -5 (b) 0  
(c) 3 (d) 5

16. Let

$$M = \begin{pmatrix} 1 & 1+i & 2i & 9 \\ 1-i & 3 & 4 & 7-i \\ -2i & 4 & 5 & i \\ 9 & 7+i & -i & 7 \end{pmatrix}. \text{ Then}$$

- (a) M has purely imaginary eigen values  
(b) M is not diagonalizable  
(c) M has eigen values which are neither real nor purely imaginary  
(d) M has only real eigen values

17. Which one of the following group is simple?

- (a)  $S_3$  (b)  $GL(2, R)$   
(c)  $Z_2 \times Z_2$  (d)  $A_5$

18. Let M denote the set of all  $2 \times 2$  matrices over the reals. Addition and multiplication on M are as follows:

$A = (a_{ij})$  and  $B = (b_{ij})$ , then  $A+B = (c_{ij})$ , where

$c_{ij} = a_{ij} + b_{ij}$ , and  $A \cdot B = (d_{ij})$  where

$d_{ij} = a_{ij} b_{ij}$ . Then which one of the following

is valid for  $(M, +, \cdot)$ ?

- (a) M is a field  
(b) M is an integral domain which is not a field  
(c) M is a commutative ring which is not an integral domain  
(d) M is a non-commutative ring

19. In  $(\mathbb{Z}, +)$ ,  $n\mathbb{Z}$  denotes the subgroup of all integral multiples of n. If  $P\mathbb{Z} = j\mathbb{Z} \cap k\mathbb{Z}$ , then

what is P equal to ?

- (a) j k (b) j + k  
(c) LCM of j & k (d) GCD of j & k

20. Which one of the following group is cyclic ?

- (a)  $\mathbb{Z}_{12} \times \mathbb{Z}_9$  (b)  $\mathbb{Z}_{10} \times \mathbb{Z}_{85}$   
(c)  $\mathbb{Z}_4 \times \mathbb{Z}_{25} \times \mathbb{Z}_6$  (d)  $\mathbb{Z}_{22} \times \mathbb{Z}_{21} \times \mathbb{Z}_{65}$

21. Let  $G = \{n \in \mathbb{Z} : 1 \leq n \leq 55, \gcd(n, 56) = 1\}$  be a multiplicative group modulo 56.

Consider the sets

$$S_1 = \{1, 9, 17, 25, 33, 41\} \text{ and}$$

$$S_2 = \{1, 15, 29, 43\}$$

which one of the following is TRUE ?

- (a)  $S_1$  is a subgroup of G but  $S_2$  is NOT a subgroup of G.  
(b)  $S_1$  is NOT a subgroup of G but  $S_2$  is a subgroup of G.  
(c) Both  $S_1$  &  $S_2$  subgroups of G  
(d) Neither  $S_1$  nor  $S_2$  is a subgroup of G.

22. Consider the alternating group

$$A_4 = \{\tau \in S_4 : \tau \text{ is an even permutation}\}$$

which of the following is FALSE ?

- (a)  $A_4$  has 12 elements  
(b)  $A_4$  has exactly one subgroup of order 4.  
(c)  $A_4$  has a subgroup of order 6.  
(d) Number of 3 cycles in  $A_4$  is 8.

23. G is a group of order 51. Then which one of the following statement is false ?

- (a) All proper subgroups of G are cyclic  
(b) If G has only one subgroup of order 3 and only one subgroup of order 17, then G is cyclic  
(c) G must have an element of order 17.  
(d) If G is abelian then there exists no proper subgroup H of G such that product of all elements of H is Identity.

24. Let  $M_3(\mathbb{R})$  be the ring of all  $3 \times 3$  real matrices.

If  $I, J \subseteq M_3(\mathbb{R})$  are defined as

$$I = \left\{ \begin{pmatrix} a & b & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\},$$

$$J = \left\{ \begin{pmatrix} a & 0 & 0 \\ b & 0 & 0 \\ c & 0 & 0 \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\},$$

- then,
- (a)  $I$  is a right ideal and  $J$  a left ideal  
 (b)  $I$  and  $J$  are both left ideals  
 (c)  $I$  and  $J$  are both right ideals  
 (d)  $I$  is a left ideal and  $J$  a right ideal
25. Let  $F_4, F_8$  and  $F_{16}$  be finite fields of 4, 8 and 16 elements respectively. Then,  
 (a)  $F_4$  is isomorphic to a subfield of  $F_8$   
 (b)  $F_8$  is isomorphic to a subfield of  $F_{16}$   
 (c)  $F_4$  is isomorphic to a subfield of  $F_{16}$   
 (d) none of the above
26. Let  $G$  be the group with the generators  $a$  and  $b$  given by  

$$G = \langle a, b : a^4 = b^2 = 1, ba = a^{-1}b \rangle$$
  
 If  $Z(G)$  denotes the centre of  $G$ , then  $G / Z(G)$  is isomorphic to  
 (a) the trivial group  
 (b)  $C_2$ , the cyclic group of order 2  
 (c)  $C_2 \times C_2$  (d)  $C_4$
27. Let  $I$  denote the ideal generated by  $x^4 + x^3 + x^2 + x + 1$  in  $Z_2[x]$  and  $F = Z_2[x]/I$ . Then,  
 (a)  $F$  is an infinite field  
 (b)  $F$  is a finite field of 4 elements  
 (c)  $F$  is a finite field of 8 elements  
 (d)  $F$  is a finite field of 16 elements
28. Let  $R = \{r_0 + r_1i + r_2j + r_3k : r_0, r_1, r_2, r_3 \in Z_3\}$  be the ring of quaternions over  $Z_3$ , where  $i^2 = j^2 = k^2 = ijk = -1$ ;  $ij = -ji = k$ ;  $jk = -kj = i$ ;  $ki = -ik = j$ . Then,  
 (a)  $R$  is a field  
 (b)  $R$  is a division ring  
 (c)  $R$  has zero divisors  
 (d) none of the above
29. Let  $a, b, c \in \mathbb{Z}$  be integers. Consider the polynomial  $p(x) = x^5 + 12ax^3 + 34bx + 43c$ .  
 (a)  $p(x)$  is irreducible over  $\mathbb{R}$  if and only if  $p(x)$  is reducible over  $\mathbb{C}$ .  
 (b)  $p(x)$  is irreducible over  $\mathbb{R}$  if and only if  $p(x)$  is irreducible over  $\mathbb{Q}$ .  
 (c)  $p(x)$  is irreducible over  $\mathbb{Z}$  if and only if  $p(x)$  is irreducible over  $\mathbb{Q}$ .  
 (d)  $p(x)$  is irreducible over  $\mathbb{Q}$  if and only if  $p(x)$  is irreducible over  $\mathbb{C}$ .
30. The number of ideals in the ring  $\frac{\mathbb{R}[x]}{(x^2 - 1)}$  is  
 (a) 1 (b) 2  
 (c) 3 (d) 4
31. A cyclic group of order 60 has  
 (a) 12 generators (b) 15 generators  
 (c) 16 generators (d) 20 generators
32. Which of the following is false?  
 (a) Any abelian group of order 27 is cyclic.  
 (b) Any abelian group of order 14 is cyclic.  
 (c) Any abelian group of order 21 is cyclic.  
 (d) Any abelian group of order 30 is cyclic.
33. Consider the following statements:  
 S1) Every group of prime order must be cyclic.  
 S2) Every group of prime order must be abelian.  
 S3) Every group of prime order has only one subgroup other than itself.  
 Which of the following is always true?  
 (a) S1 and S2 are true but S3 is false.  
 (b) S2 and S3 are true but S1 is false.  
 (c) S1 and S3 are true but S2 is false.  
 (d) S1, S2 and S3 are true.  
 (e) None of the above.
34. The number of solutions of  $X^5 \equiv 1 \pmod{163}$  in  $\mathbb{Z}_{163}$  is  
 (a) 1 (b) 2  
 (c) 3 (d) 4
35. Pick out the correct statements from the following list:  
 (a) A homomorphic image of a UFD (unique factorization domain) is again a UFD.  
 (b) The element  $2 \in \mathbb{Z}[\sqrt{-5}]$  is irreducible in  $\mathbb{Z}[\sqrt{-5}]$ .  
 (c) Units of the ring  $\mathbb{Z}[\sqrt{-5}]$  are the units of  $\mathbb{Z}$ .  
 (d) the element 2 is a prime element in  $\mathbb{Z}[\sqrt{-5}]$ .
36. Pick out the true statement(s):  
 (a) The set of all  $2 \times 2$  matrices with rational entries (with the usual operations of matrix addition and matrix multiplication) is a ring which has no nontrivial ideals.



- (b) Let  $R=C[0,1]$  be considered as a ring with the usual operations of pointwise addition and pointwise multiplication. Let  $I = \{f : [0,1] \rightarrow \mathbb{R} \mid f(1/2) = 0\}$ . Then  $I$  is a maximal ideal.
- (c) Let  $R$  be a commutative ring and let  $P$  be a prime ideal of  $R$ . Then  $R/P$  is an integral domain.
37. Pick out the cases where the given subgroup  $H$  is a normal subgroup of the group  $G$ .
- (a)  $G$  is the group of all  $2 \times 2$  invertible upper triangular matrices with real entries, under matrix multiplication, and  $H$  is the subgroup of all such matrices  $(a_{ij})$  such that  $a_{11}=1$ .
- (b)  $G$  is the group of all  $2 \times 2$  invertible upper triangular matrices with real entries, under matrix multiplication, and  $H$  is the subgroup of all such matrices  $(a_{ij})$  such that  $a_{11}=a_{22}$ .
- (c)  $G$  is the group of all  $n \times n$  invertible matrices with real entries, under matrix multiplication, and  $H$  is the subgroup of such matrices with positive determinant.
38. Pick out the cases where the given ideal is a maximal ideal.
- (a) The ideal  $15\mathbb{Z}$  in  $\mathbb{Z}$ .
- (b) The ideal  $I = \{f : f(0) = 0\}$  in the ring  $C[0,1]$  of all continuous real valued functions on the interval  $[0, 1]$ .
- (c) The ideal generated by  $x^3 + x + 1$  in the ring of polynomials  $\mathbb{F}_3[x]$ , where  $\mathbb{F}_3$  is the field of three elements.
39. Which of the following integral domains are Euclidean domains?
- (a)  $\mathbb{Z}[\sqrt{-3}] = \{a + b\sqrt{-3} : a, b \in \mathbb{Z}\}$
- (b)  $\mathbb{Z}[x]$
- (c)  $\mathbb{R}[x^2, x^3] = \left\{ f = \sum_{i=0}^n a_i x^i \in \mathbb{R}[x] : a_1 = 0 \right\}$
- (d)  $\left( \frac{\mathbb{Z}[x]}{(2, x)} \right)[y]$  where  $x, y$  are independent variables and  $(2, x)$  is the ideal generated by 2 and  $x$
40. Let  $R = \mathbb{Q}[x]/I$  where  $I$  is the ideal generated by  $1 + x^2$ . Let  $y$  to the coset of  $x$  in  $R$ . Then
- (a)  $y^2 + 1$  is irreducible over  $R$ .
- (b)  $y^2 + y + 1$  is irreducible over  $R$ .
- (c)  $y^2 - y + 1$  is irreducible over  $R$ .
- (d)  $y^3 + y^2 + y + 1$  is irreducible over  $R$ .
41. For the linear programming problem
- Maximize  $z = x_1 + 2x_2 + 3x_3 - 4x_4$
- Subject to  $2x_1 + 3x_2 - x_3 - x_4 = 15$
- $6x_1 + x_2 + x_3 - 3x_4 = 21$
- $8x_1 + 2x_2 + 3x_3 - 4x_4 = 30$
- $x_1, x_2, x_3, x_4 \geq 0$
- $x_1 = 4, x_2 = 3, x_3 = 0, x_4 = 2$  is
- (a) an optimal solution
- (b) a degenerate basic feasible solution
- (c) a non-degenerate basic feasible solution
- (d) a non-basic feasible solution
42. Which one of the following is TRUE ?
- (a) Every linear programming problem has a feasible solution.
- (b) If a linear programming problem has an optimal solution then it is unique.
- (c) The union of two convex sets is necessarily convex.
- (d) Extreme points of the disk  $x^2 + y^2 \leq 1$  are the points on the circle  $x^2 + y^2 = 1$ .
43. The bilinear transformation  $w = 2Z/(Z - 2)$  maps  $\{Z : |Z - 1| < 1\}$  onto
- (a)  $\{w : \operatorname{Re} w < 0\}$  (b)  $\{w : \operatorname{Im} w > 0\}$
- (c)  $\{w : \operatorname{Re} w > 0\}$  (d)  $\{w : |w + 2| < 1\}$
44. Let  $f : \mathbb{C} \rightarrow \mathbb{C}$  be a complex valued function of the form  $f(x, y) = u(x, y) + iv(x, y)$ , Suppose that  $u(x, y) = 3x^2y$ . Then
- (a)  $f$  cannot be holomorphic on  $\mathbb{C}$  for any choice of  $v$ .
- (b)  $f$  is holomorphic on  $\mathbb{C}$  for a suitable choice of  $v$ .
- (c)  $f$  is holomorphic on  $\mathbb{C}$  for all choices of  $v$ .
- (d)  $u$  is not differentiable

45. Let  $I_r = \int_{C_r} \frac{dz}{z(z-1)(z-2)}$ ,

where  $C_r = \{z \in \mathbb{C} : |z| = r\}$ ,  $r > 0$ .

Then

(a)  $I_r = -2\pi i$  if  $r \in (2, 3)$

(b)  $I_r = \frac{1}{2}$  if  $r \in (0, 1)$

(c)  $I_r = -2\pi i$  if  $r \in (1, 2)$

(d)  $I_r = 0$  if  $r > 3$

46. Let  $f_n, g_n : [0, 1] \rightarrow \mathbb{R}$  be defined by

$$f_n(x) = x^2(1-x^2)^{n-1} \text{ and}$$

$$g_n(x) = \frac{1}{n^2(1+x^2)} \text{ for } n \in \mathbb{N}.$$

Then, on  $[0, 1]$ ,

(a)  $\sum_{n=1}^{\infty} f_n(x)$  converges uniformly but not

$$\sum_{n=1}^{\infty} g_n(x)$$

(b)  $\sum_{n=1}^{\infty} g_n(x)$  converges uniformly but not

$$\sum_{n=1}^{\infty} f_n(x)$$

(c) both  $\sum_{n=1}^{\infty} f_n(x)$  and  $\sum_{n=1}^{\infty} g_n(x)$  converge uniformly

(d) neither  $\sum_{n=1}^{\infty} f_n(x)$  nor  $\sum_{n=1}^{\infty} g_n(x)$  converges uniformly

47. Let  $S = \{0\} \cup \left\{ \frac{1}{4n+7} : n = 1, 2, \dots \right\}$ . Then the

number of analytic functions which vanish only on  $S$  is

(a) infinite (b) 0

(c) 1 (d) 2

48. Consider the function

$$f(x) = \cos(|x-5|) + \sin(|x-3|) + |x+10|^5 - (|x|-4)^2$$

At which of the following points is  $f$  not differentiable?

(a)  $x = 5$  (b)  $x = 3$

(c)  $x = -10$  (d)  $x = 0$

49. Which of the following is/are true?

(a)  $\log \frac{x+y}{2} \leq \frac{\log x + \log y}{2}$  for all  $x, y > 0$ .

(b)  $e^{\frac{x+y}{2}} \leq \frac{e^x + e^y}{2}$  for all  $x, y > 0$ .

(c)  $\sin \frac{x+y}{2} \leq \frac{\sin x + \sin y}{2}$  for all  $x, y > 0$ .

(d)  $\frac{(x+y)^k}{2^k} \leq \max\{x^k, y^k\}$  for all  $x, y > 0$

and all  $k \geq 1$ .

50. Which of the following is/are correct?

(a)  $\left(1 + \frac{1}{n}\right)^{n-1} \rightarrow e$  as  $n \rightarrow \infty$

(b)  $\left(1 - \frac{1}{n+1}\right)^n \rightarrow e$  as  $n \rightarrow \infty$ .

(c)  $\left(1 + \frac{1}{n}\right)^{n^2} \rightarrow e$  as  $n \rightarrow \infty$ .

(d)  $\left(1 - \frac{1}{n^2}\right)^n \rightarrow e$  as  $n \rightarrow \infty$ .

## CLASS SCHEDULE

The classroom coaching will be conducted at Mukherjee Nagar (New Delhi) Centre. The duration of the entire course is 5 months. This particular course is punctuated for catering the need of students who are going to appear for CSIR-NET. For those aspirants who will appear for GATE examination, their course will also be finished within the same time frame as they will get ample time for revision. The classes will be held in live environment keeping an eye on each and every individual/ aspirant so that they can get the best out of it. The interacting classroom session will have its repercussions manifold. The timings for the class and other requisite things will be facilitated to the aspirants at the time of admission.

### **Salient Features of Classroom Coaching**

- ❖ Daily 4hrs. class
- ❖ Proper strategy and guidance in answer writing skill with the short-cuts.
- ❖ To the point and very effective study material.
- ❖ Proper coverage, conceptual and in depth analysis of the syllabus.
- ❖ Individual interaction and feedback.
- ❖ Every week one module-wise test equipped with doubt session, feedback and one-to-one interaction.
- ❖ Final Mock test series will be conducted for the revision.
- ❖ Time-to-Time interaction with successful candidates.

## ADMISSION PROCEDURE

The aspirants who want to be the part of our institute, they can take admission by depositing Rs. 15000/- as registration fee to book your seat either in cash or through demand draft (DD) in favour of IMS (institute of mathematical sciences) payable at New Delhi. The balance fee must be submitted atleast on or before 3 days the commencement of classes. If it is possible, you can also deposit full fee through DD/Cash. Seats are limited, therefore the candidates are requested to take admission as soon as possible and reserve their seat in advance within the stipulated time so that they should not have to face any problem in admission. Once the seats will be filled, no admission will be entertained.



Institute for IAS/ IFOs/ CSIR/ GATE Examinations

**Contact Nos.**

**011-45629987, 09999329111, 09999197625**

Email: [ims4ims2010@gmail.com](mailto:ims4ims2010@gmail.com) || Website: [ims4maths.com](http://ims4maths.com)

**The following Book List is Being Recommended**

by **IMS**

**NAME OF THE TOPIC**

**NAME OF THE AUTHOR**

**Unit-1**

- |                  |  |
|------------------|--|
| - Real Analysis  | (i) H.L. Royden                                |
|                  | (ii) S.C. Malik and Savita Arora               |
| - Linear Algebra | (i) Hoffman and Kunze                          |
|                  | (ii) Schaum Series (3000 Solved Problems Book) |

**Unit-2**

- |                    |                       |
|--------------------|-----------------------|
| - Complex Analysis | (i) Schaum Series     |
|                    | (ii) R.V. Churchill   |
| - Algebra          | (i) Joseph A. Gallian |
|                    | (ii) I.N. Herstein    |
| - Topology         | (i) G.F. Simmons      |

**Unit-3**

- |                                   |                         |
|-----------------------------------|-------------------------|
| - Ordinary Differential Equations | (i) G.F. Simmons        |
|                                   | (ii) M.D. Rai Singhania |
| - Partial Differential Equations  | (i) Ian. Sneddon        |
|                                   | (ii) M.D. Rai Singhania |
| - Numerical Analysis              | (i) Jain and Iyengar    |
| - Calculus of Variations          | (i) A.S. Gupta          |
| - Linear Integral Equations       | (i) Shanti Swarup       |
| - Classical Mechanics             | (i) Gupta Kumar Sharma  |

**Unit-4**

- |              |                |
|--------------|----------------|
| - Statistics | (i) S.C. Gupta |
|--------------|----------------|

# COURSES OFFERED

by

# IMS



## Mathematics CSIR/GATE

● Regular Course/Online ● Correspondance/ Postal Course ● Test Series

### Other Courses Offered by IMS

- GATE: Engineering Mathematics Module
- CSIR: Engineering Sciences (Mathematics and Engineering Aptitude Related) Module
- IAS/IFoS (Mathematics)
- TIFR (Tata Institute of Fundamental Research)
- ISI (Indian Statistical Institute) Delhi/ Kolkata/ Bangalore
- NBHM (National Board of Higher Mathematics)

### Faculty Members

Our institute has faculty with proven track record of their academics and long teaching experience. Their expertise in their fields gives a sound platform to the aspiring candidates to create a holistic environment for these coveted examinations.

**1 K. Venkanna**  
(13 years teaching experience)

**2 Dr. Srinivas (Retd.)**  
(Asso. Professor)

**3 A. Sridevi**  
(NET Qualified)

**4 G. Aruna**  
(M.Sc. B.Ed. GATE Qualified)



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