IFoS 2010

Sl. No. 8951 B–JGT–K–NBB

MATHEMATICS

Paper—II

Time Allowed : Three Hours	Maximum Marks : 200

INSTRUCTIONS

Candidates should attempt Question Nos. 1 and 5 which are compulsory, and any THREE of the remaining questions, selecting at least ONE question from each Section.

All questions carry equal marks.

The number of marks carried by each part of a question is indicated against each.

Answers must be written in ENGLISH only.

Assume suitable data, if considered necessary, and indicate the same clearly.

Symbols and notations have their usual meanings, unless indicated otherwise.

Section—A

1. Answer any *four* parts from the following :

(a) Let

$$G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \middle| \, a \in \mathbb{R}, \, a \neq 0 \right\}$$

Show that G is a group under matrix multiplication. 10

(b) Let F be a field of order 32. Show that the only subfields of F are F itself and $\{0, 1\}$. 10

(c) If $f : \mathbb{R} \to \mathbb{R}$ is such that

$$f(x+y) = f(x) f(y)$$

for all x, y in $\mathbb{R}$ and $f(x) \neq 0$ for any x in $\mathbb{R}$, show that $f'(x) = f(x)$ for all x in $\mathbb{R}$ given that $f'(0) = f(0)$ and the function is differentiable for all x in $\mathbb{R}$. 10

(d) Determine the analytic function $f(z) = u + iv$ if $v = e^x(x \sin y + y \cos y)$. 10

(e) A captain of a cricket team has to allot four middle-order batting positions to four batsmen. The average number of runs scored by each batsman at these positions are as follows. Assign each batsman his batting position for *maximum* performance : 10

Batsman \ Batting position	IV	V	VI	VII
A	40	25	20	35
B	36	30	24	40
C	38	30	18	40
D	40	23	15	33

2. (a) A rectangular box open at the top is to have a surface area of 12 square units. Find the dimensions of the box so that the volume is maximum. 13

(b) Prove or disprove that $(\mathbb{R}, +)$ and $(\mathbb{R}^+, \cdot)$ are isomorphic groups where $\mathbb{R}^+$ denotes the set of all positive real numbers. 13

(c) Using the method of contour integration, evaluate

$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)^2(x^2+2x+2)}$$ 14

3. (a) Show that zero and unity are only idempotents of Z_n if $n = p^r$, where p is a prime. 13

(b) Evaluate

$$\iint_R (x - y + 1)\, dx\, dy$$

where R is the region inside the unit square in which $x + y \geq \frac{1}{2}$. 13

(c) Solve the following linear programming problem by the simplex method : 14

Maximize $Z = 3x_1 + 4x_2 + x_3$

subject to

$$x_1 + 2x_2 + 7x_3 \leq 8$$
$$x_1 + x_2 - 2x_3 \leq 6$$
$$x_1, x_2, x_3 \geq 0$$

4. *(a)* Let R be a Euclidean domain with Euclidean valuation d. Let n be an integer such that $d(1)+n \geq 0$. Show that the function $d_n : R-\{0\} \to S$, where S is the set of all negative integers defined by $d_n(a) = d(a)+n$ for all $a \in R-\{0\}$ is a Euclidean valuation. 13

(b) Obtain Laurent's series expansion of the function

$$f(z) = \frac{1}{(z+1)(z+3)}$$

in the region $0 < |z+1| < 2$. 13

(c) ABC Electricals manufactures and sells two models of lamps, L_1 and L_2, the profit per unit being Rs 50 and Rs 30, respectively. The process involves two workers W_1 and W_2, who are available for 40 hours and 30 hours per week, respectively. W_1 assembles each unit of L_1 in 30 minutes and that of L_2 in 40 minutes. W_2 paints each unit of L_1 in 30 minutes and that of L_2 in 15 minutes. Assuming that all lamps made can be sold, determine the weekly production figures that maximize the profit. 14

Section—B

5. Answer any *four* parts from the following :

(a) Find the general solution of

$$x(y^2 + z)p + y(x^2 + z)q = z(x^2 - y^2)$$ 10

(b) Solve $x \log_{10} x = 1 \cdot 2$ by regula falsi method. 10

(c) Convert the following : 10

(i) $(736 \cdot 4)_8$ to decimal number

(ii) $(41 \cdot 6875)_{10}$ to binary number

(iii) $(101101)_2$ to decimal number

(iv) $(AF63)_{16}$ to decimal number

(v) $(101111011111)_2$ to hexadecimal number

(d) Show that the sum of the moments of inertia of an elliptic area about any two tangents at right angles is always the same. 10

(e) A two-dimensional flow field is given by $\psi = xy$. Show that—

(i) the flow is irrotational;

(ii) ψ and ϕ satisfy Laplace equation.

Symbols ψ and ϕ convey the usual meaning. 10

6. *(a)* Using Lagrange interpolation, obtain an approximate value of $\sin(0\cdot15)$ and a bound on the truncation error for the given data : 12

$$\sin(0\cdot1) = 0\cdot09983, \ \sin(0\cdot2) = 0\cdot19867$$

(b) Draw a flow chart for finding the roots of the quadratic equation $ax^2 + bx + c = 0$. 12

(c) Solve

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$$

given the conditions

(i) $u(0, t) = u(\pi, t) = 0, \quad t > 0$

(ii) $u(x, 0) = \sin 2x, \quad 0 < x < \pi$ 16

7. *(a)* Find the general solution of

$$(D - D' - 1)(D - D' - 2)z = e^{2x-y} + \sin(3x + 2y)$$

13

(b) Show that $\phi = (x - t)(y - t)$ represents the velocity potential of an incompressible two-dimensional fluid. Further show that the streamlines at time t are the curves

$$(x - t)^2 - (y - t)^2 = \text{constant}$$ 13

(c) Find the interpolating polynomial for (0, 2), (1, 3), (2, 12) and (5, 147). 14

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8. *(a)* A mass m_1, hanging at the end of a string, draws a mass m_2 along the surface of a smooth table. If the mass on the table be doubled, the tension of the string is increased by one-half. Show that $m_1 : m_2 = 2 : 1$. 13

(b) Solve the initial value problem

$$\frac{dy}{dx} = \frac{y - x}{y + x}, \quad y(0) = 1$$

for $x = 0 \cdot 1$ by Euler's method. 13

(c) Show that the vorticity vector $\vec{\Omega}$ of an incompressible viscous fluid moving under no external forces satisfies the differential equation

$$\frac{D\vec{\Omega}}{Dt} = (\vec{\Omega} \cdot \nabla)\vec{q} + \nu \nabla^2 \vec{\Omega}$$

where ν is the kinematic viscosity. 14

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