

UPSC-IAS MAIN-2013 EXPECTED NEW PATTERN

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



Test held on
8-SEPT.-2013

TEST SERIES (MAIN)-2013 (QUALITY IMPROVEMENT PROGRAMME)

Test Code: QIP(M) IAS / Test- 07

MATHEMATICS (PAPER-I) FULL LENGTH

by **K. VENKANNA**

The person with 14 years of Teaching Experience

Time: Three Hours

Maximum Marks: 250

INSTRUCTIONS

1. This question paper-cum-answer booklet has 60 pages and has **33 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed.

Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

**DO NOT WRITE ON
THIS SPACE**

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
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5	(a)			
	(b)			
	(c)			
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	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION - A**Question No. 1**

- (a). Show that the set $P(t) = \{at^2 + bt + c/a, b, c \in \mathbb{R}\}$ forms a vector space over the field \mathbb{R} . Find a basis for this vector space. What is the dimension of this vector space? (10)

- (b) If the product of two non-zero square matrices is a zero matrix, show that both of them must be singular matrices. (10)

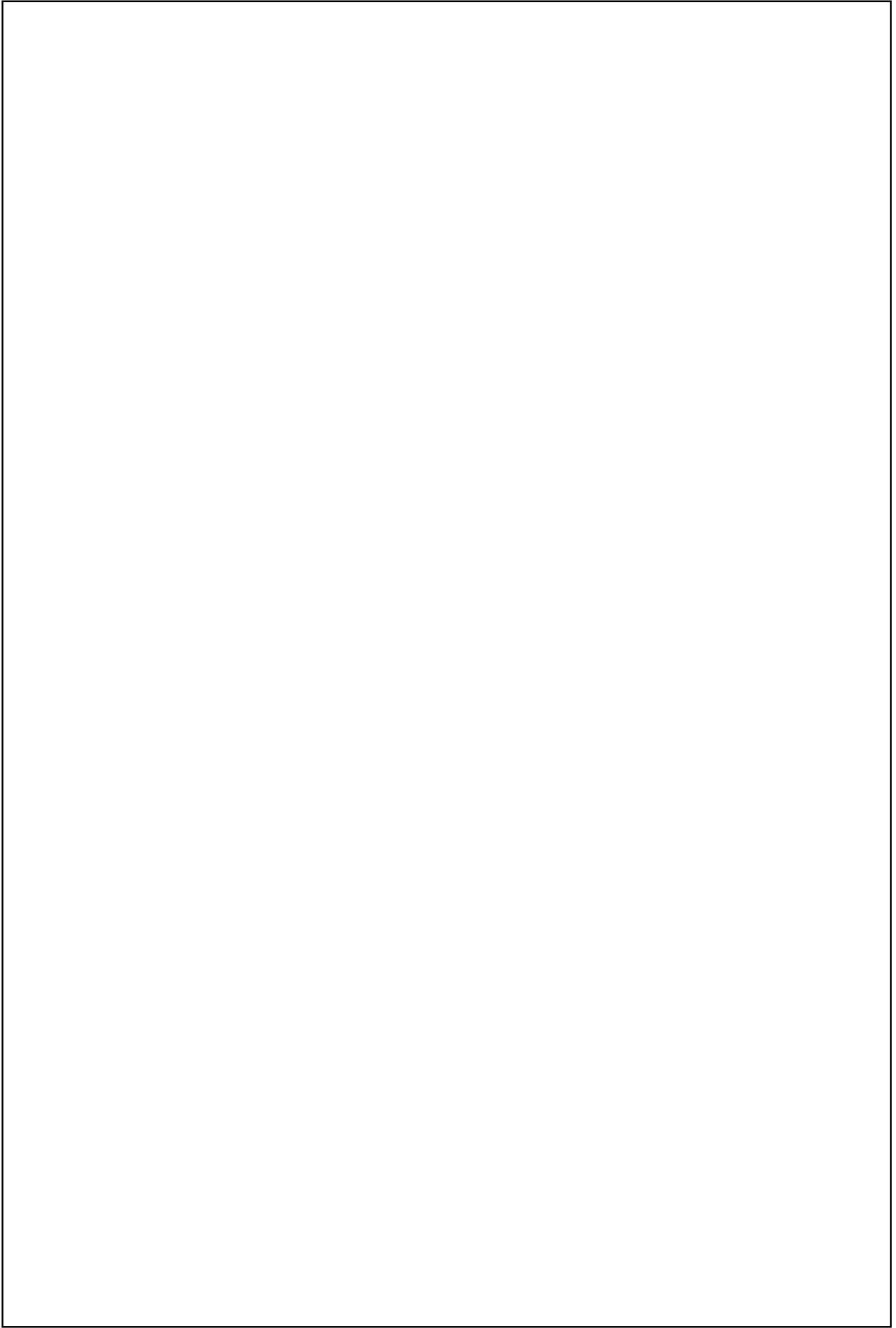
- (c) Discuss the continuity of the function $f(x,y) = \begin{cases} \frac{2xy^2}{x^3 + 3y^3} & ;(x,y) \neq (0,0) \\ 0 & ;(x,y) = (0,0) \end{cases}$ (10)

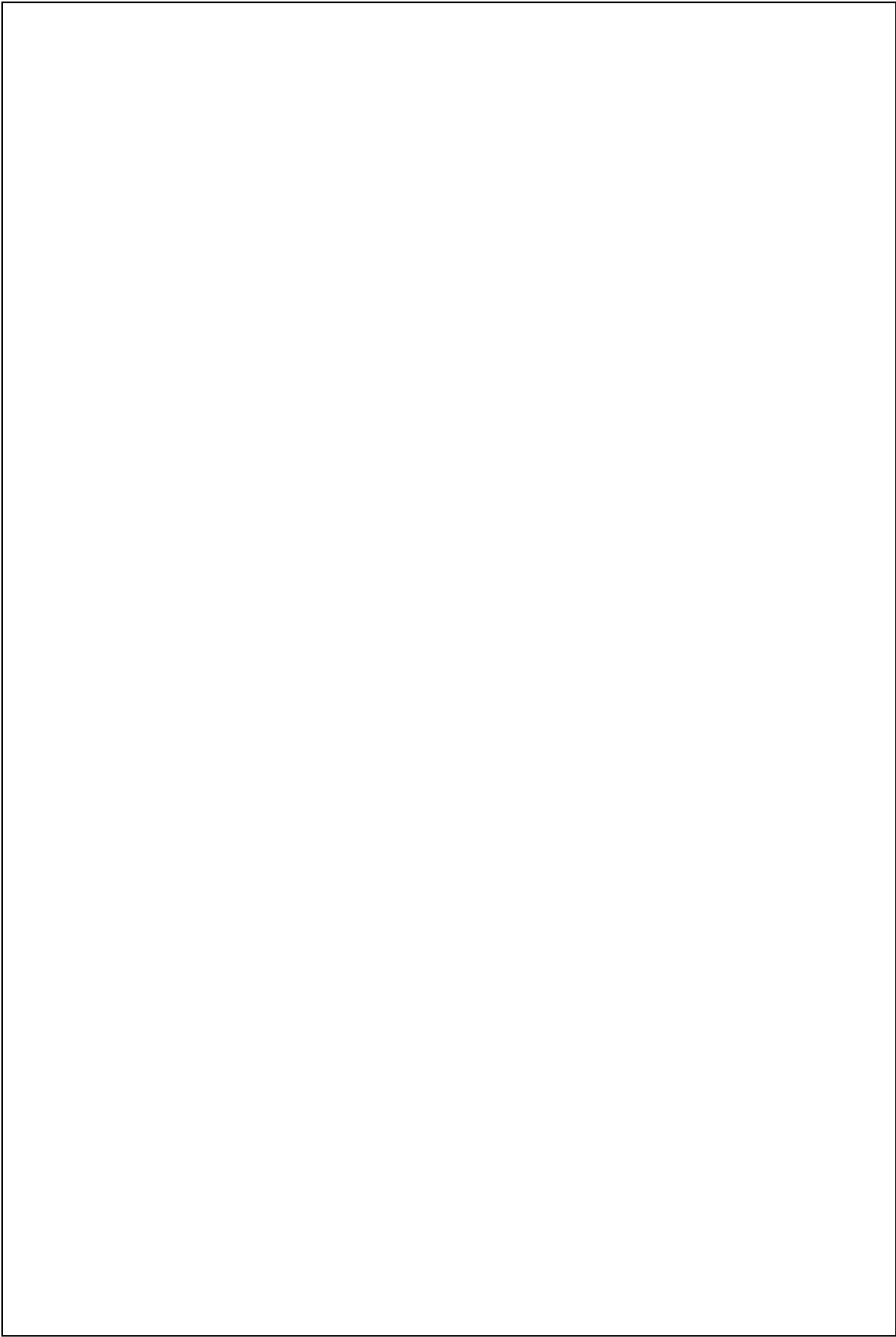
(d) Using Taylor's theorem, show that $1 + x + \frac{x^2}{2} < e^x < 1 + x + \frac{x^2 e^x}{2}, x > 0.$ (10)

- (e) Show that the length of the shortest distance between the line $z = x \tan \alpha, y = 0$ and any tangent to the ellipse $x^2 \sin^2 \alpha + y^2 = a^2, z = 0$ is constant. (10)

Question No. 2

- (a) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalizable. Also find the diagonal form and diagonalizing matrix P. (15)





- (b) Let A be a 3×3 upper triangular matrix with real entries. If $a_{11} = 1, a_{22} = 2$ and $a_{33} = 3$, determine α, β and γ such that $A^{-1} = \alpha A^2 + \beta A + \gamma I$. (10)

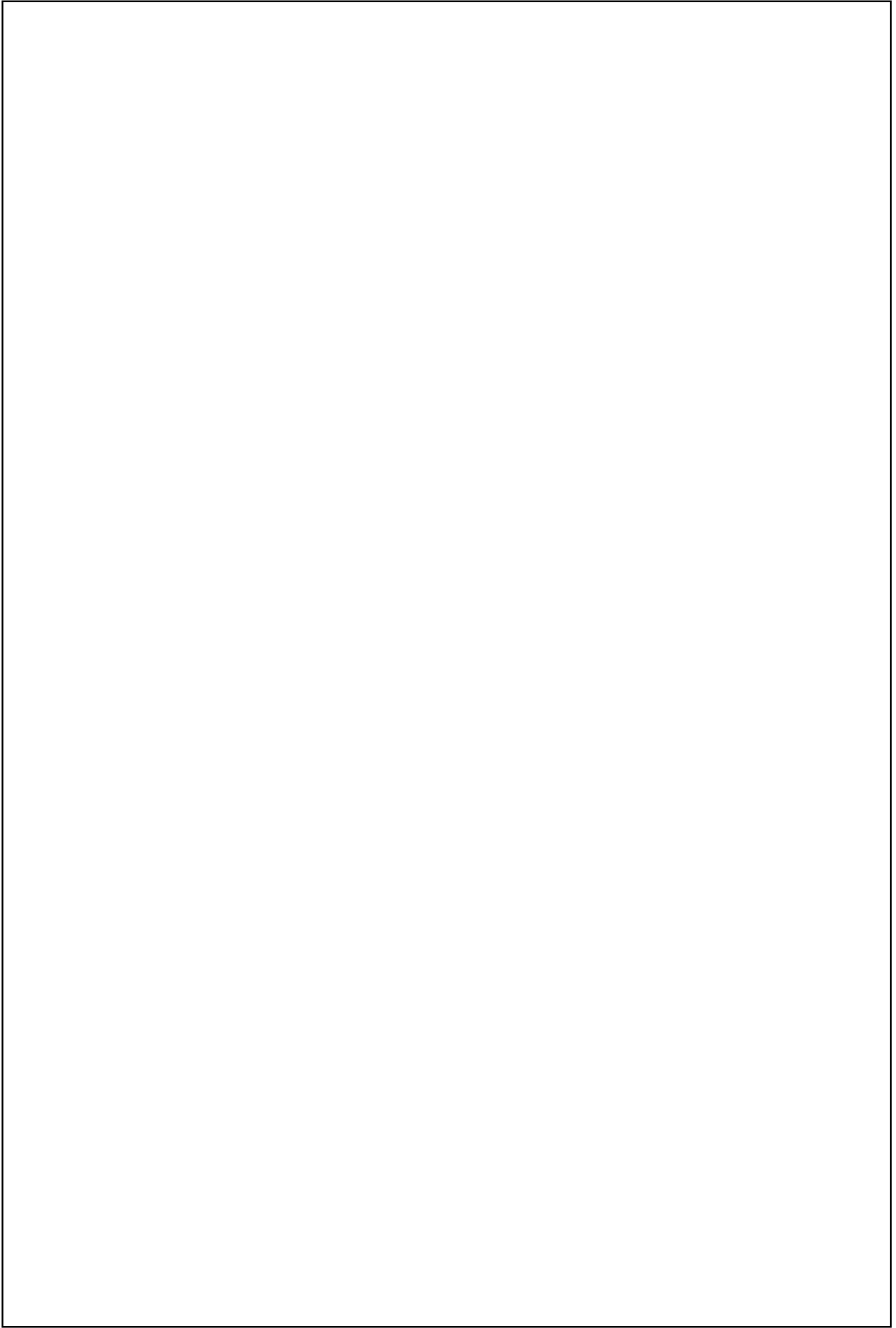
(c) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be such that

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & , \text{if } x < 0 \\ c & , \text{if } x = 0 \\ \frac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}} & , \text{if } x > 0 \end{cases}$$

Determine the values of a, b, c for which the function is continuous at $x = 0$.

(10)

- (d) A sphere of constant radius r passes through the origin O and cuts the axes in A, B, C . Find the locus of the foot of the perpendicular from O to the plane ABC . (15)



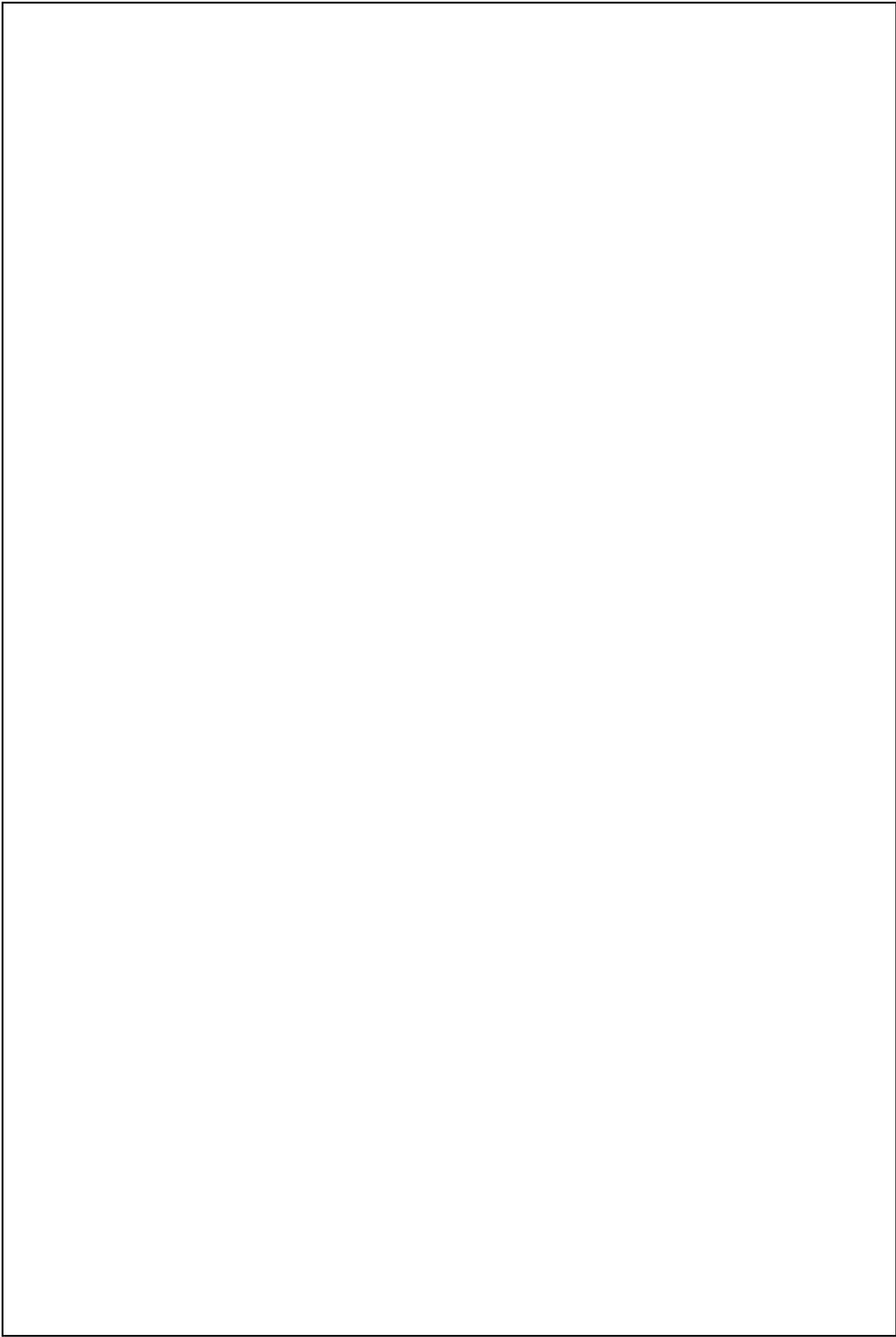
Question No. 3

- (a) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be defined by $T(x, y, z) = (y + z, z, 0)$. Show that T is a linear transformation. If $V \in \mathbb{R}^3$ is such that $T^2(V) \neq 0$, then show that $B = \{V, T(V), T^2(V)\}$ forms a basis of \mathbb{R}^3 . Compute the matrix of T with respect to B. Also find a $V \in \mathbb{R}^3$ such that $V^2(V) \neq 0$. (15)

(b) If $z = (x + y)\phi(y/x)$, where ϕ is any arbitrary function. Prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$. (10)

(c) Evaluate the integral $\int_0^\infty \int_0^x x e^{-x^2/y} dx dy$ by changing the order integration. (12)

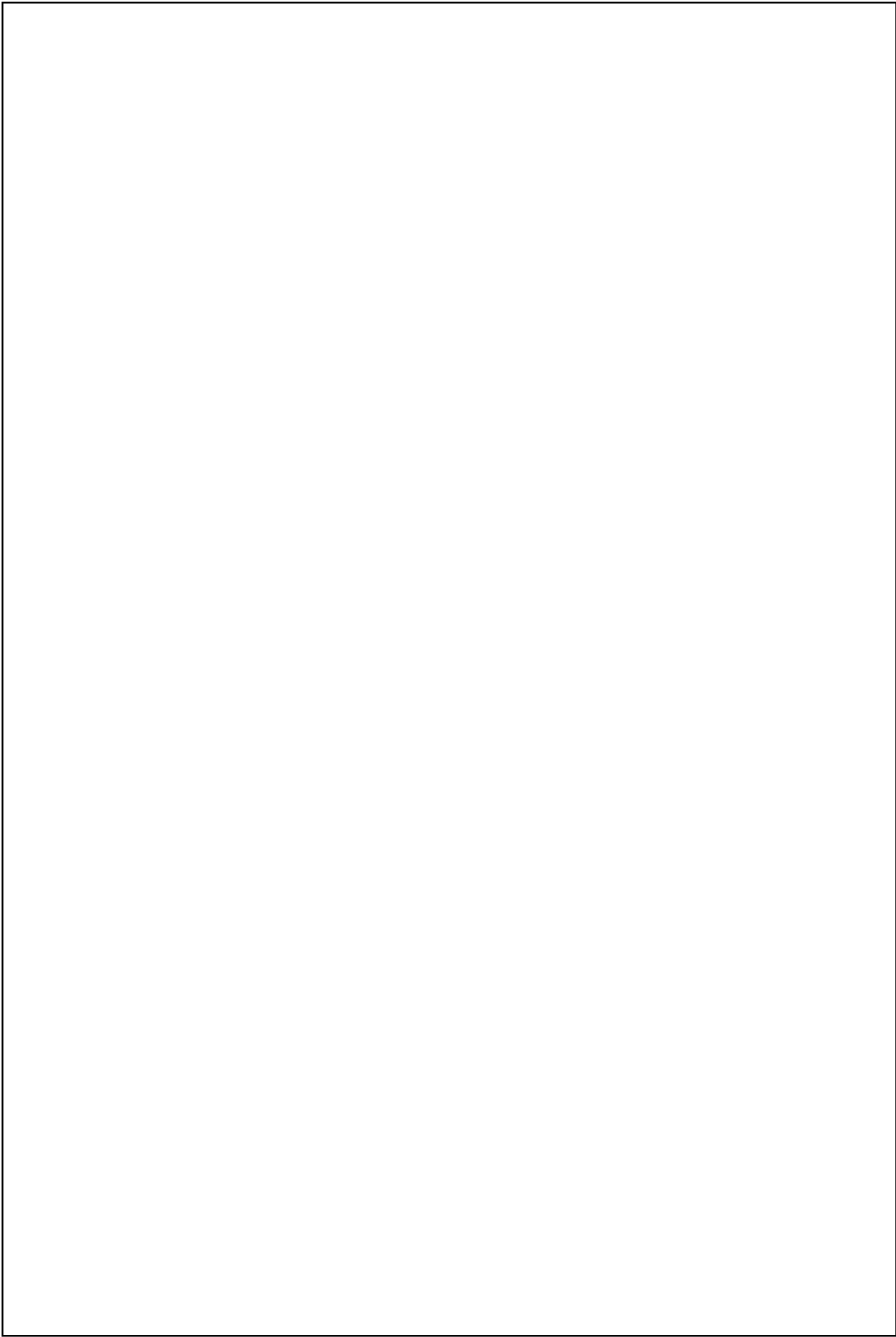
- (d) Enveloping cylinders of the quadric $ax^2 + by^2 + cz^2 = 1$ meet the plane $z = 0$ in rectangular hyperbola; show that the central perpendiculars to their planes of contact generate the cone $b^2cx^2 + a^2cy^2 + ab(a+b)z^2 = 0$. (13)

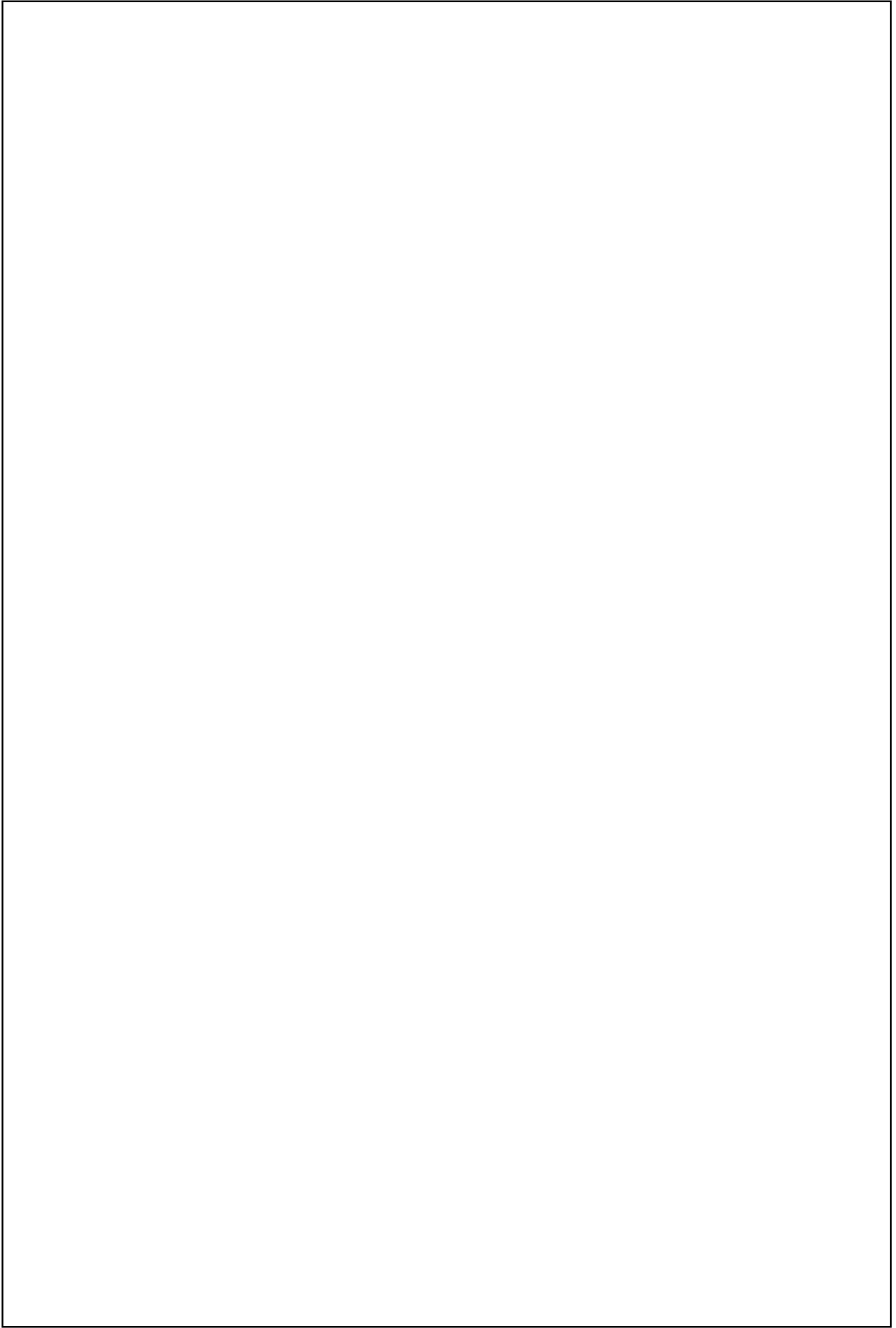


Question No. 4

(a) Let V be the vector space of polynomials over \mathbf{R} . Let U and W be the subspaces generated by $\{t^3 + 4t^2 - t + 3, t^3 + 5t^2 + 5, 3t^3 + 10t^2 - 5t + 5\}$ and $\{t^3 + 4t^2 + 6, t^3 + 2t^2 - t + 5, 2t^3 + 2t^2 - 3t + 9\}$ respectively. Find

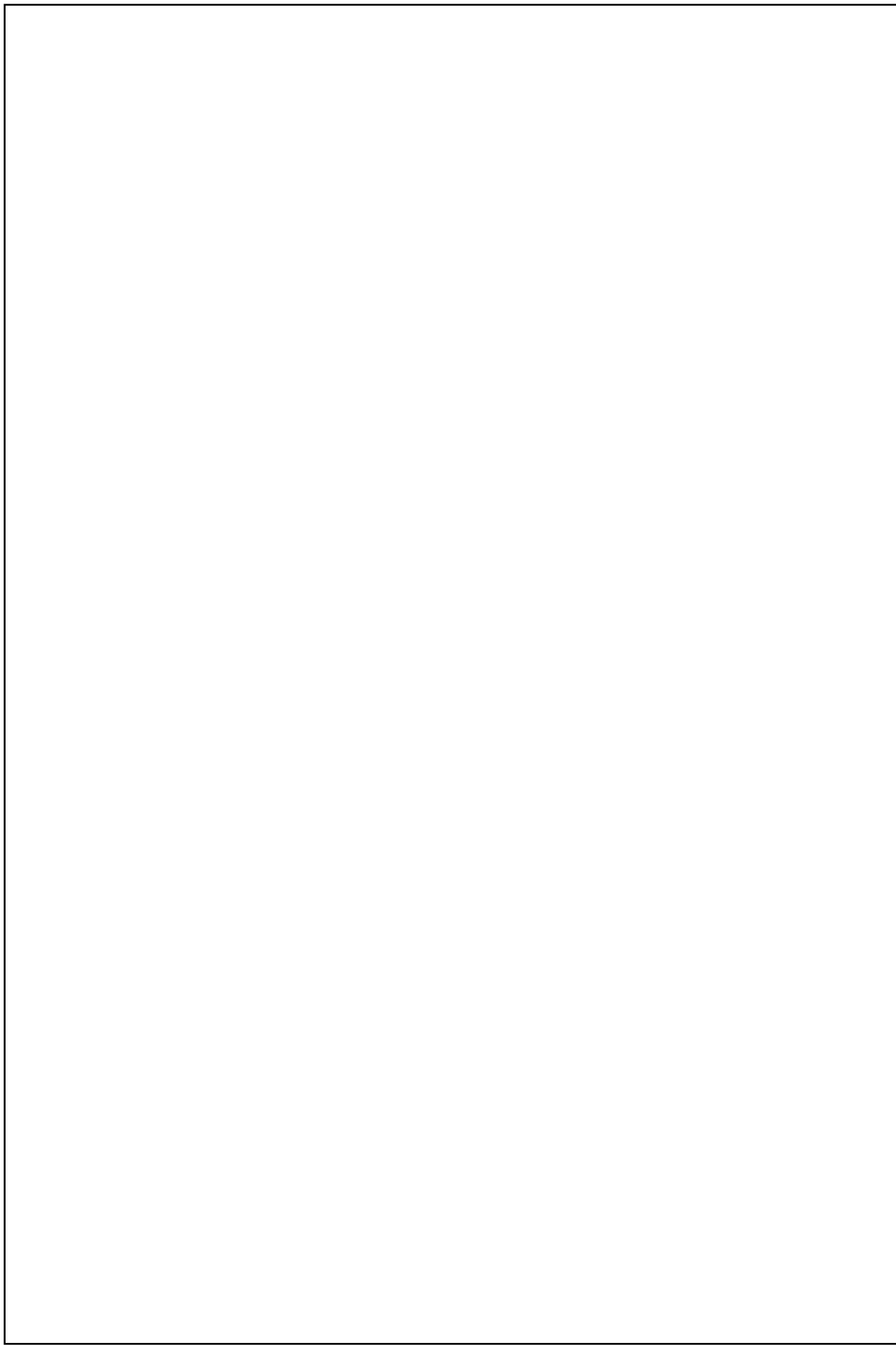
(i) $\dim(U + W)$ (ii) $\dim(U \cap W)$. (17)

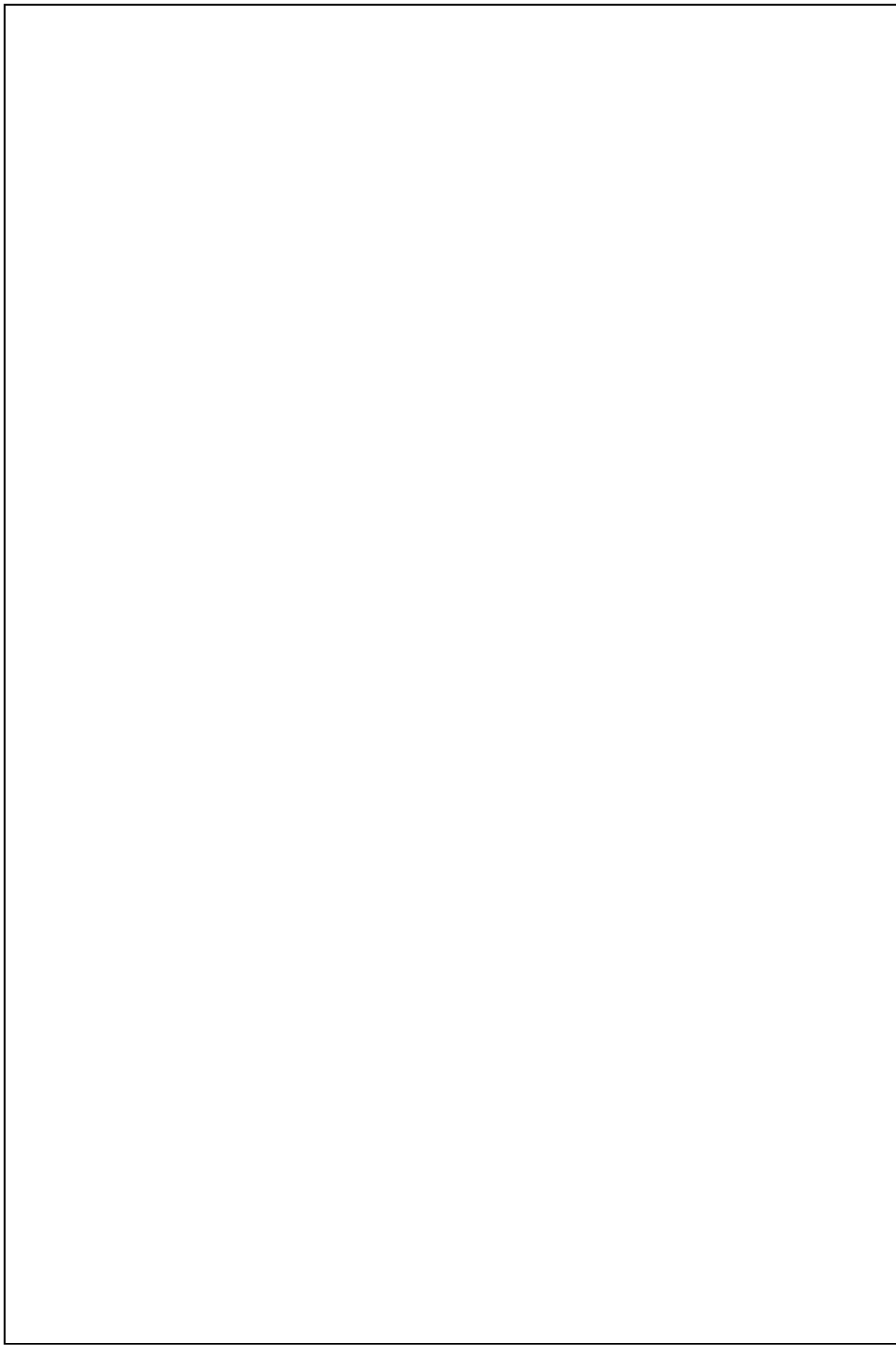




- (b) Find the volume bounded above by the sphere $x^2 + y^2 + z^2 = 2a^2$ and below by the paraboloid $az = x^2 + y^2$.
(16)

(c) Show that the feet of normals from the point (α, β, γ) on the paraboloid $x^2 + y^2 = 2az$ lie on a sphere. (16)





SECTION - B**Question No. 5**

- (a). Show that the only curves having constant curvature are circles and straight lines. (10)

(b) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2 + \gamma} + \frac{y^2}{b^2 + \gamma} = 1$, where λ is a parameter. (10)

- (c) A heavy string of uniform density and thickness is suspended from two given points in the same horizontal plane. A weight, an n th that of the string, is attached to its lowest point; show that if θ, ϕ be the inclinations to the vertical of the tangents at the highest and lowest points of the string $\tan \phi = (1 + n) \tan \theta$. **(10)**

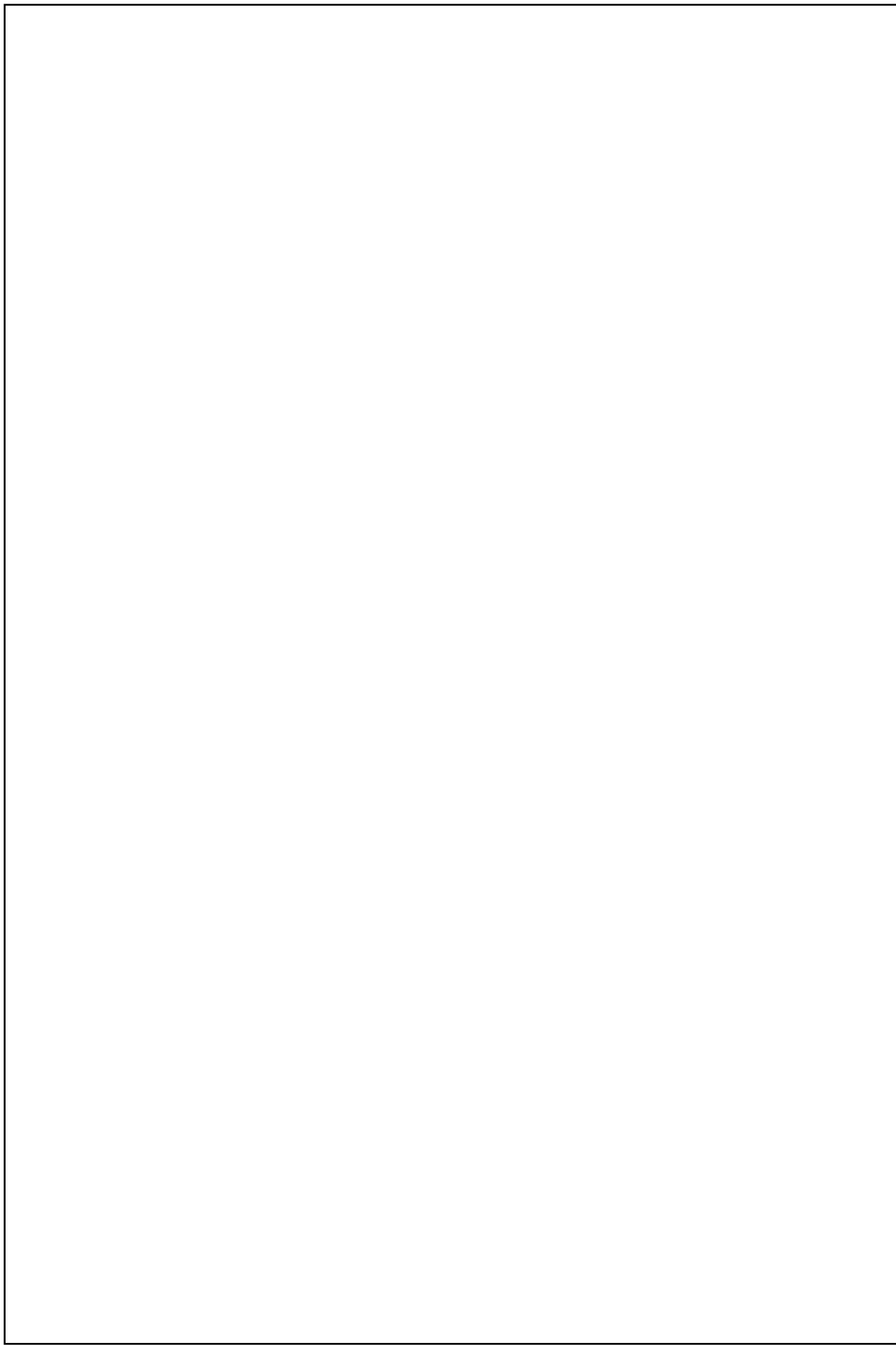
- (d) A body of mass (m_1+m_2) moving in a straight line is split into two parts of masses m_1 and m_2 by an internal explosion which generates kinetic energy E . Show that if after the explosion the two parts move in the

same line as before, their relative speed is $\sqrt{\left[\frac{2E(m_1 + m_2)}{m_1 m_2}\right]}$. (10)

- (e) The acceleration of a particle at any time $t \geq 0$ is given by $a = \frac{dv}{dt} = 12 \cos 2t \mathbf{i} - 8 \sin 2t \mathbf{j} + 16t \mathbf{k}$. If the velocity \mathbf{v} and displacement \mathbf{r} are zero at $t = 0$, find \mathbf{v} and \mathbf{r} at any time. (10)

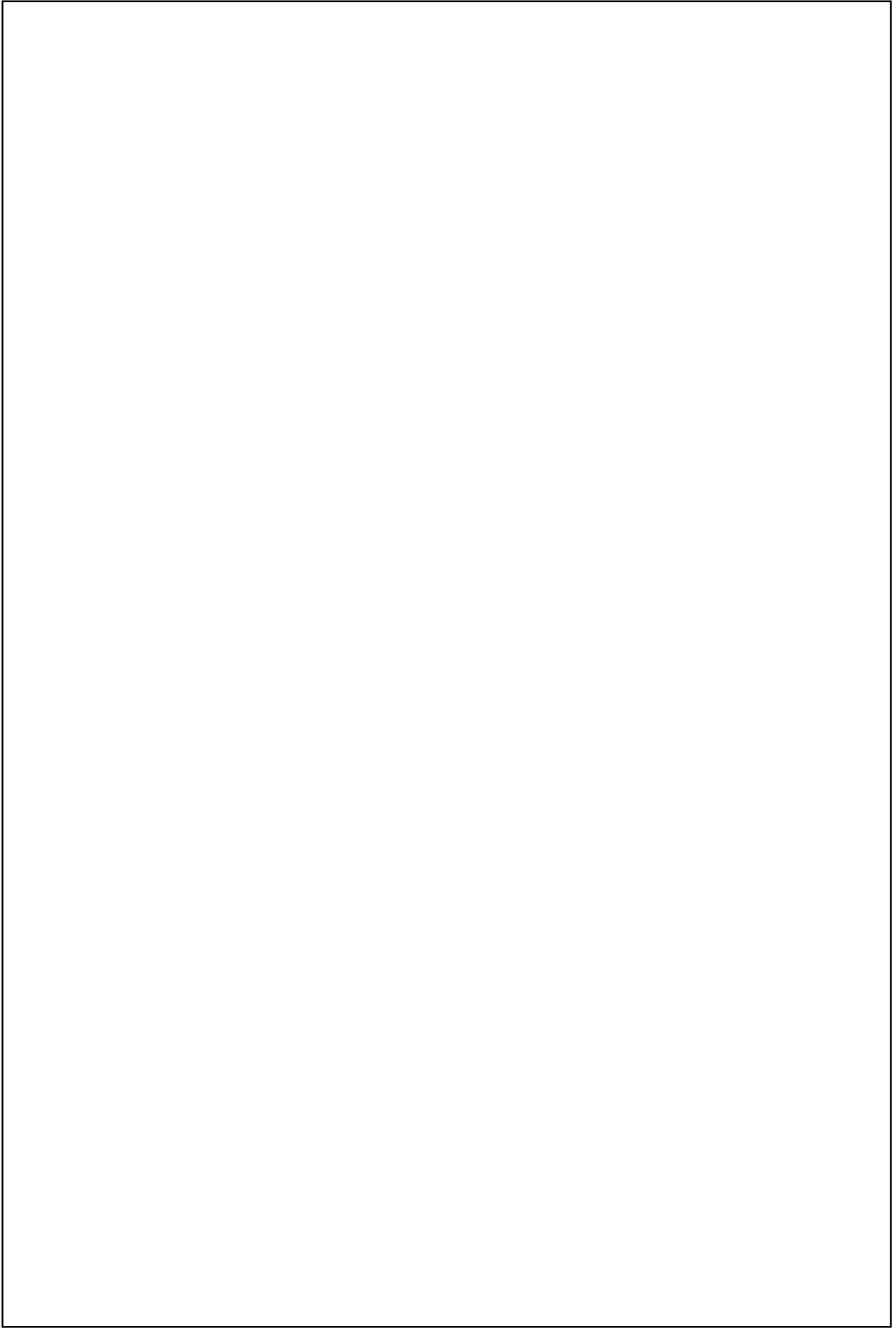
Question No. 6

- (a) Using Laplace transforms, solve the initial value problem $(D^4 + 2D^2 + 1)y = 0$,
 $y(0) = 0, y'(0) = 1, y''(0) = 2$ and $y'''(0) = -3$. (10)



(b) Solve $(1+2x)^2 \left(\frac{d^2 y}{dx^2} \right) - 6(1+2x) \left(\frac{dy}{dx} \right) + 16y = 8(1+2x)^2$, $y(0) = 0, y'(0) = 2$. (14)

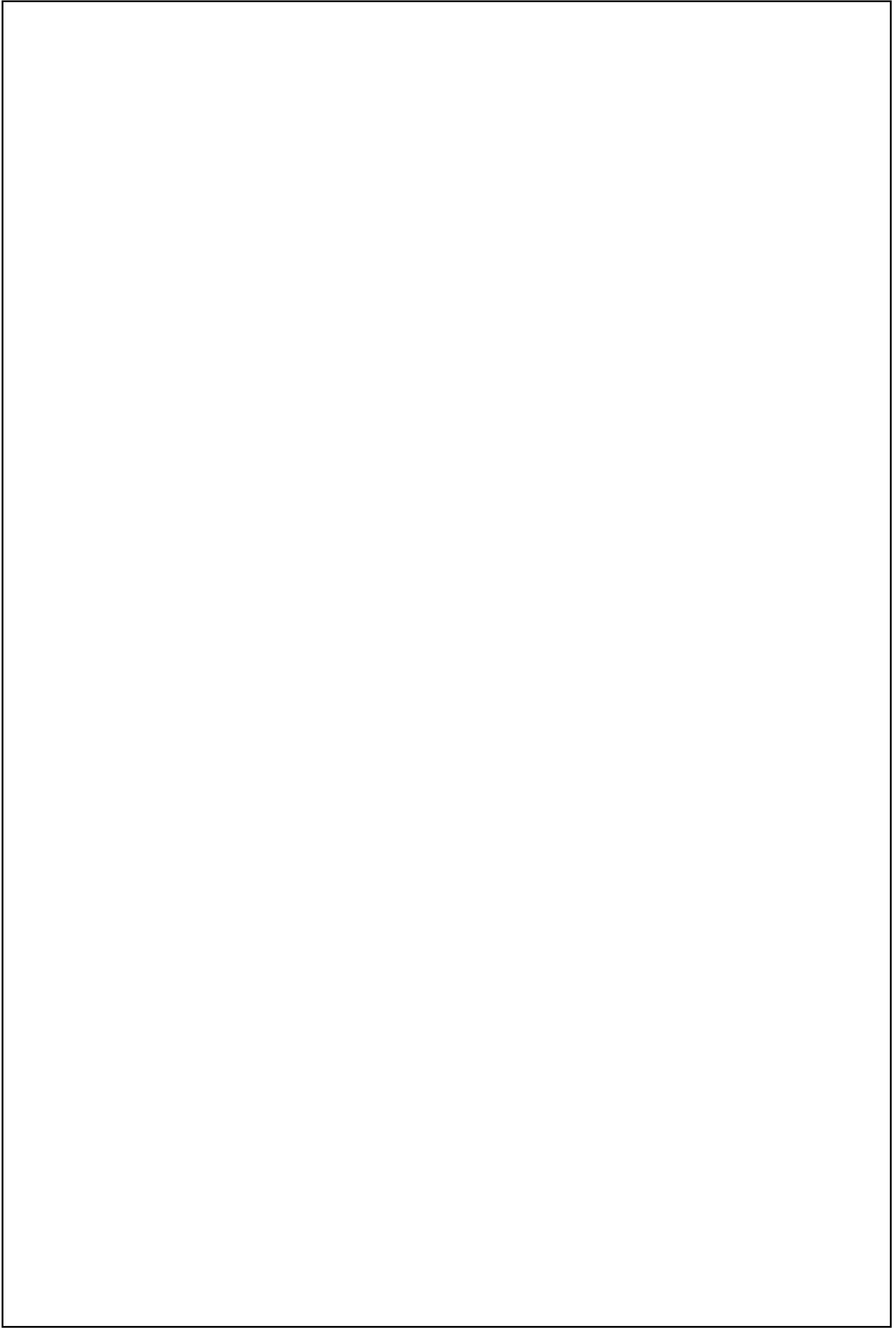
- (c) Reduce the equation $y^2(y - xp) = x^4 p^2$ to Clairaut's form and hence solve the equation. (12)



(d) Solve by using the method of variation of parameters $dy^2/dx^2 - 2(dy/dx) = e^x \sin x$. (14)

Question No. 7

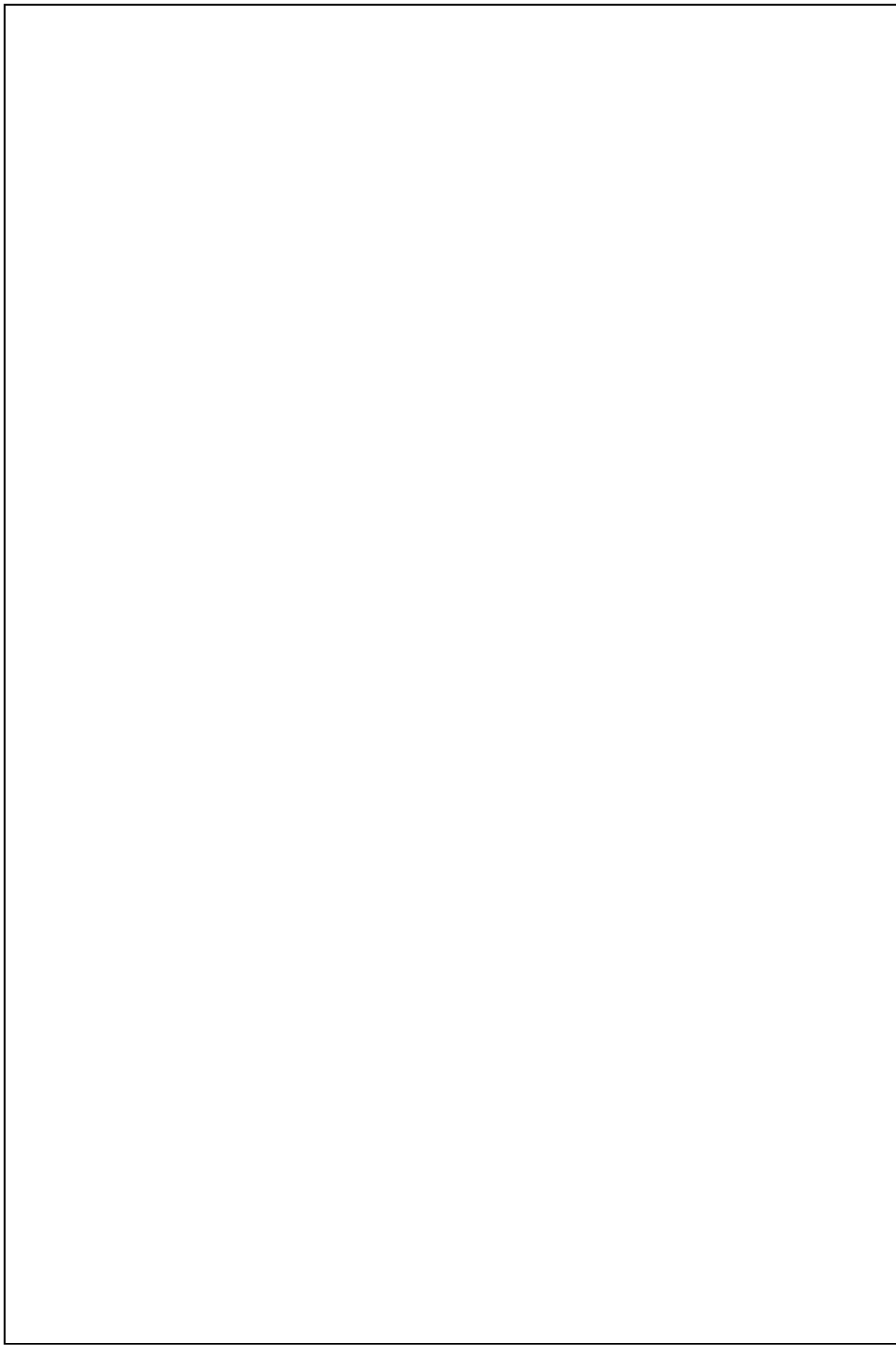
- (a) A endless chain of weight W rests in the form of a circular band round a smooth vertical cone which has its vertex upwards. Find the tension in the chain due to its weight, assuming the verticle angle of the cone to be 2α . (13)



- (b) A thin hemispherical bowl, of radius b and weight W rests in equilibrium on the highest point of a fixed sphere, of radius a , which is rough enough to prevent any sliding. Inside the bowl is placed a small smooth sphere of weight w ; show that the equilibrium is not stable unless $w < W \frac{a-b}{2b}$. (12)

- (c) A battleship is steaming ahead with velocity u . A gun is mounted on the ship so as to point straight backwards and is set at an angle of elevation α . If v be the velocity of projection relative to the gun, show that the range is $(2v/g)\sin\alpha(v\cos\alpha - u)$, and the angle of elevation for maximum range is

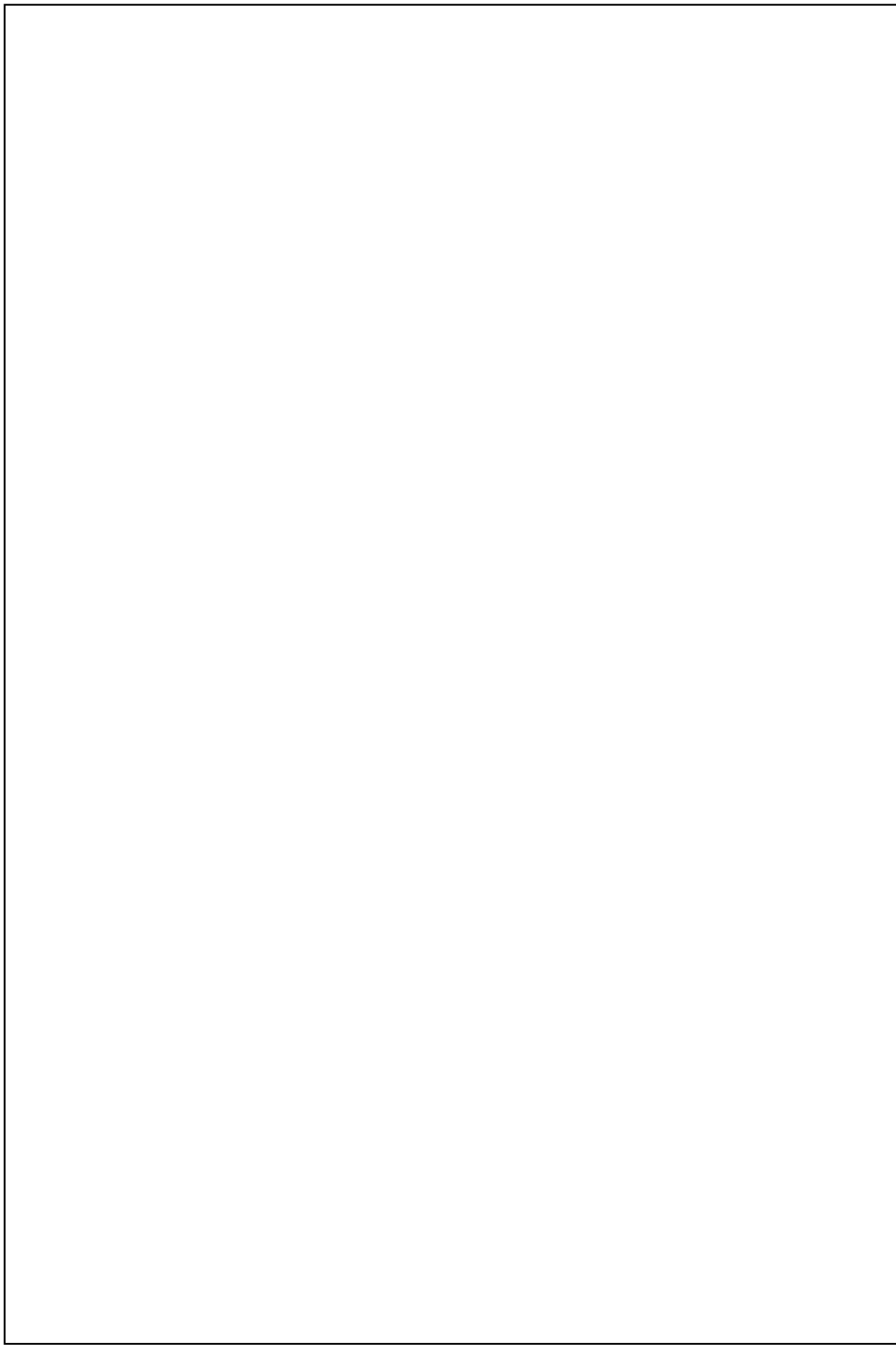
$$\cos^{-1} \left[\frac{u + \sqrt{(u^2 + 8v^2)}}{4v} \right]. \quad (12)$$



- (d) Show that the only law for a central attraction for which the velocity in a circle at any distance is equal to the velocity acquired in falling from infinity to the distance is that of inverse cube. (13)

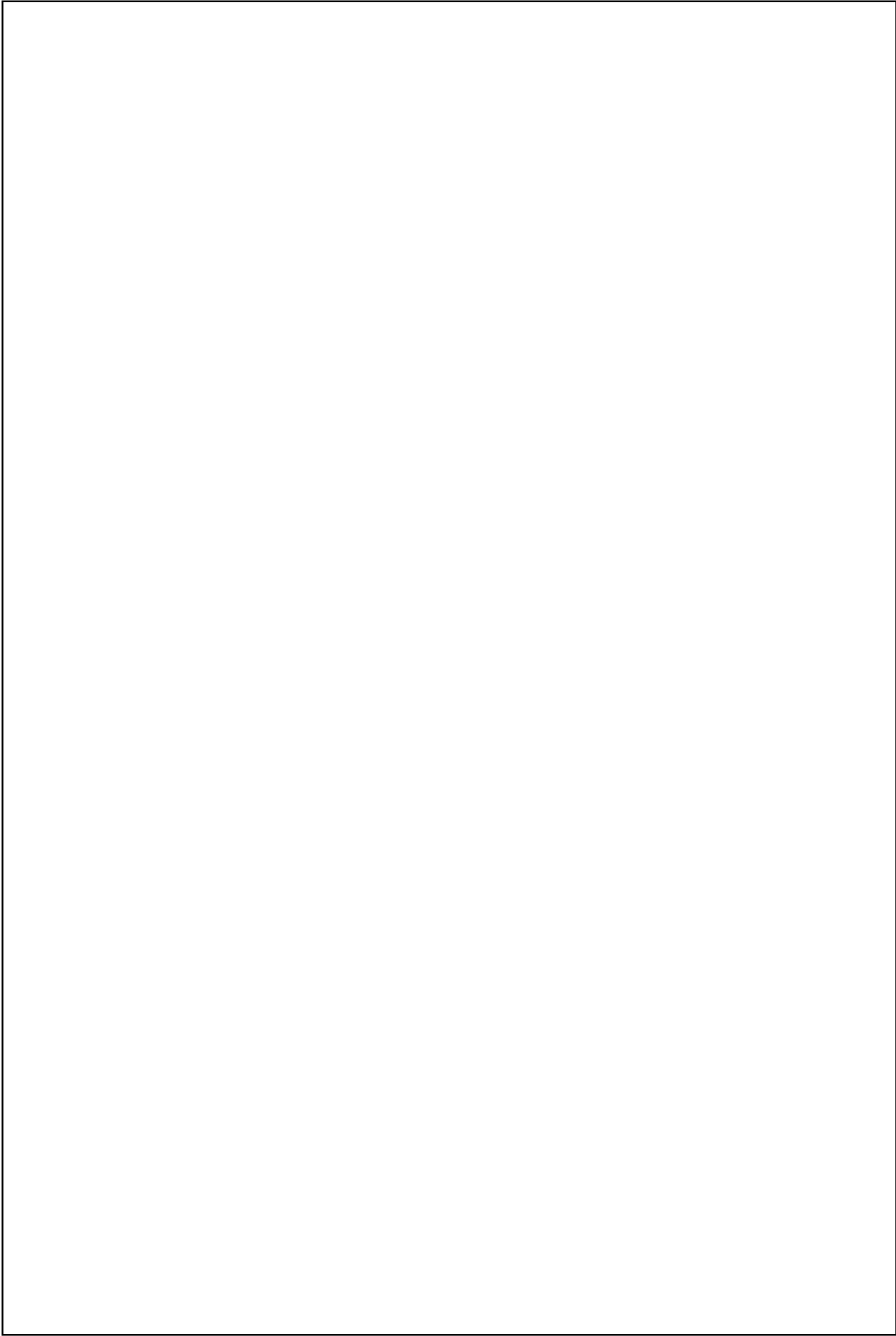
Question No. 8

- (a) Find the values of constants a, b, c so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at $(1, 2, -1)$ has a maximum magnitude 64 in a direction parallel to z -axis. (6)

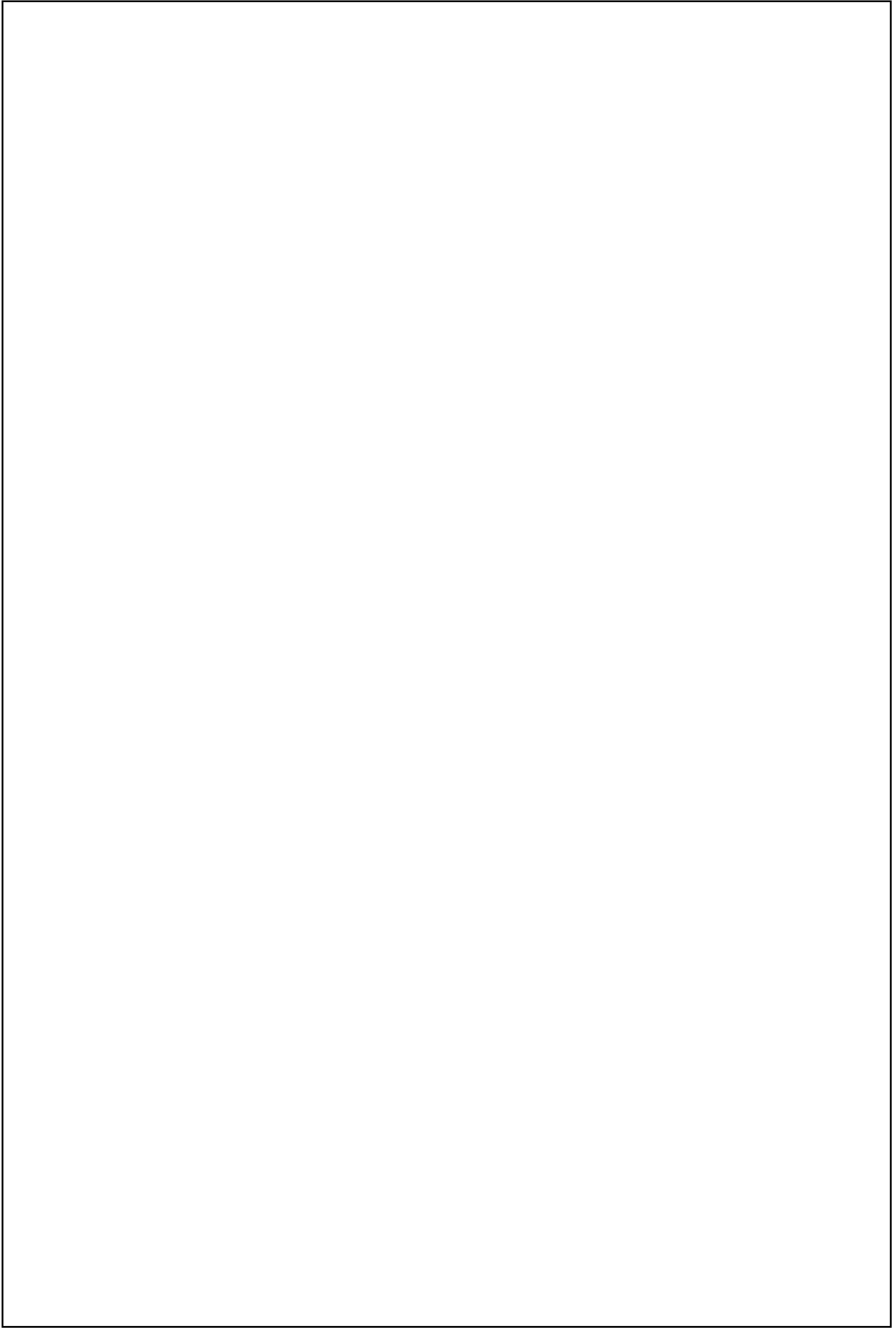


(b) If $\nabla^2 f(r) = 0$ show that $f(r) = c_1 \log r + c_2$ where $r^2 = x^2 + y^2$ and c_1, c_2 are arbitrary constants. (10)

- (c) Apply Green's theorem in the plane to evaluate $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$, where C is the boundary of the surface enclosed by the x-axis and the semi-circle $y = (1 - x^2)^{1/2}$. (14)



- (d) Verify Stoke's theorem for $\mathbf{F} = (2x - y) \mathbf{i} - yz^2 \mathbf{j} - y^2z \mathbf{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (20)



ROUGH SPACE

ROUGH SPACE

ROUGH SPACE

END OF THE EXAMINATION

FEEDBACK with Students Testimonial

RAMESH RANJAN
AIR-76 (IAS-2012)

Respected Venkanna sir
I am deeply indebted to you in getting success in civil service exam-2012 & getting AIR-76.

Your paper guidance throughout the whole process helped me in analyzing the weak points of my preparation. The test cards helped me a lot only because of your meticulously designed question paper & solution etc.

I am also very much thankful to you for the hard written booklet for the various topics of mathematics.

I suggest students to take marks as an optional in this new pattern for fetching good marks under the guidance of Venkanna sir. Sir has started 95 classes also, so I would advice the aspirants that they may join that class also, cause his guidance is immensely good. I will continue to appreciate you in future also for any help.

Ramesh Ranjan
AIR-76
Civil Service-2012

ANKIT VERMA
AIR-247 (IAS-2012)

Respected Venkanna Sir,
I owe my success in Civil Services Examination 2012 to inspiring and practical guidance of Venkanna Sir. His guidance helped in other areas of relevance to this examination as well.

Regarding Mathematics as an optional, I would like to suggest that it keeps up your interest in the preparation especially for engineering and science background students besides being very scoring.

Ankit Verma
ANKIT VERMA
Rank 247

PRADEEP MISHRA
AIR-633 (IAS-2012)

I am grateful to Venkanna Sir for his invaluable guidance and support which immensely contributed to my success. Thank you Sir for making me understand Mathematics.

P. Mishra
(PRADEEP MISHRA)

KETAN BANSAL
AIR-655 (IAS-2012)

To
Venkanna Sir
IMS
New Delhi

Text books offered enough practice of all types of questions from each and every topic of syllabus. Detailed solutions helped in dealing complicated concepts and gave new ways to look at the problems. Conversations with Venkanna Sir over phone were helpful in clearing doubts which arose while studying at home and it also kept me motivated. I think that the text books is a must for every candidate with Maths optional.

Thanks Venkanna Sir.

Ketan Bansal
KETAN BANSAL

SANJAY KR. JAIN
AIR-667 (IAS-2012)

I want to thank Venkanna Sir for his continuous guidance and support all through my preparation of civil services. He is great mentor and guided me not just in mathematics but in other areas as well. Without his efforts and unflinching faith in me, this would not have been possible.

Sanjay Kumar Jain
Rank - 667

SANTOSH KUMAR
AIR-849 (IAS-2012)

I am very thankful to I.M.S. for Mathematics preparation for Civil Services. I joined I.M.S. last Session and within all this Test papers are very well prepared according to the new syllabus and U.P.S.C. Question paper. Test Solutions are well organized to help candidate in mentioning all important steps required to the Solution of question.
Test Series helped me a lot to crack Civil Services Exam.

Thanks
Santosh Kumar

ANUPAM SHUKLA
AIR-7 (IFS-2012)

Respected Venkanna Sir,
I owe my success in Indian Forest Service Examination 2012 to your inspiring lectures in Mathematics that evoked my interest and preparation to take challenge with confidence. Many thanks to I.M.S and Venkanna Sir
ANUPAM SHUKLA
RANK - 07
INDIAN FOREST SERVICE EXAM-2012

DILIP KR. YADAV
AIR-48 (IFS-2012)

Respected Venkanna Sir,
I am writing this letter to inform my heartfelt thanks for providing me all the supports during the course of preparation. I am very thankful to you for your coaching materials because of which I am able to secure AIR-48 in IFS-2012 Exam.

Dilip Kumar Yadav
AIR-48
IFS-2012

HIMANSHU GUPTA
AIR-7 (IAS-2011)

Respected sir
I feel very happy to inform you that I have achieved AIR-5 in IAS Exam 2011 and AIR-7 in Civil Service Exam-2011.
Sir your guidance and inputs were very important in my preparation. Test series programme especially was very important in providing a simulated examination environment, which was extremely helpful.
I strongly recommend Venkanna Sir's classes for Mathematics classes.
Thanking You
Yours sincerely
(Himanshu Gupta)
(HIMANSHU GUPTA)
AIR-5 in IAS Exam-2011
AIR-7 in CSE-2011

ARIJIT MUKHERJEE
AIR-25 (IAS-2011)

I, Arijit Mukherjee (Roll no. 202 2012) got selected in Release List declared by UPSC in Civil Service Examination 2011.
I appeared in interview for CSE 2012 as well.
Matters were some of my optionals and had joined test series for same, conducted by Venkanna Sir (Director of I.M.S.).
His guidance was helpful to me in clearing this examination.
I am expressing sincere gratitude to him.

Arijit Mukherjee

GULNEET SINGH KHURANA
AIR-220 (IAS-2011)

Respected Sir, I feel very proud in informing you that, I have made it through the civil services exam (2011). I am very thankful to you for providing your valuable guidance in this great success.

I feel very happy while informing other aspirants that I could not have made it without your guidance which is clear in my marks over the year.

Yours sincerely
Gulneet Singh
AIR 220 (IAS-2011)

AJIT PRATAP SINGH
AIR-220 (IAS-2011)

Sir, I am really thankful for providing me your valuable time. Because of that only, I scored top in mathematics. You helped me in covering the subject thoroughly. And your well prepared notes and test series has given me a way to cover the entire syllabus in time bound fashion.

Thank you.

Ajit Pratap Singh
Rank - 220 (UPSC)

MEGHA AGARWAL
AIR-538 (IAS-2011)

The preparation for civil services never was and never will be easy, but with proper guidance and consistent effort one can sail through. Mathematics is a subject liked by few and even those who like it, think a number of times before opting for it. For there is a misconception among the aspirants that science subjects do not fetch good marks. But sincerely telling, if one has any interest in this subject, especially like Venkatesh Sir from IAS (Inst. of Mathematical Sciences) notes can very well guide you and help you in keeping faith in this subject. I would thank Sir for his continuous guidance and encouraging words that helped me intellectually and emotionally. Also for the study material provided and the test series conducted were of immense help. I will continue to look towards him for any help in the future also.

MEGHA AGARWAL
CSE-2011
AIR - 538
BANGALORE

BHAGWATI PRASAD KALAL
AIR-154 (IAS-2010)

→ This year's result has proved that mathematics is also one of the most fetching subjects.

→ If proper guidance and notes are available then high scoring is also possible in mathematics.

→ In my third attempt I took guidance from Venkatesh Sir and consulted few topics from his notes. I found his help very useful and his moral support worked for me.

→ I suggest student to take maths as optional and to enjoy it.

Bhagwati
Rank - 154

ABHISHEK
AIR-223 (IAS-2010)

Mathematics has in recent years shown upward thrust in marks as compared to other optionals. With correct investment of time and energy, it can truly become a feather in an aspirant's Civil Services Preparation. I want to thank K. Venkanna Sir of IMS to give continuous support and guidance, which helped me achieve excellent marks in Mathematics.

Abhishek

AWAKASH KUMAR
AIR-276 (IAS-2010)

ANAKASH KUMAR
CSE-2010 ; Roll - 114429
RANK-276 ; TOTAL MARKS- 1176
OPTIONALS - MECHANICAL ENGINE. & MATHEMATICS.

11th May 2011, 2:17 pm, a clerk of mine to life changed. And it changed drastically for betterment. Starting the civil services is not easy. It was especially with the kind of optionals I opted for. I was advised 'in' no. of times to change one of the optionals to any of the humanities. But I persisted with those stupidly because of my interest and a firm belief that I can do better than them. Being a Mechanical Engineer I was tempted towards those optionals but I faced difficulties in some of the areas of mathematics like Modern Algebra among others. But with the kind guidance of Sir K. Venkanna, Sir of the Institute of Mathematical Sciences it became a lot easier and some sort of an amazing spark. The test series was meticulously devised which gave me a examination like feeling and it my final performance was not too different from those in the test series. The start cost of guidance and motivation provided by Sir was of immense help and I thank him. Besides my father, mother, wife & friends for this sense of achievement in my life bestowed upon me by the almighty.

Abhishek
2011/5

IMS
(INSTITUTE OF MATHEMATICAL SCIENCES)
INSTITUTE FOR IAS/IFoS/CSIR/GATE EXAMINATIONS

MATHEMATICS
(Optional)

by **K. VENKANNA** (13yrs. teaching exp.)

Our Achievements from 2008 to 2012

IAS

IFoS

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